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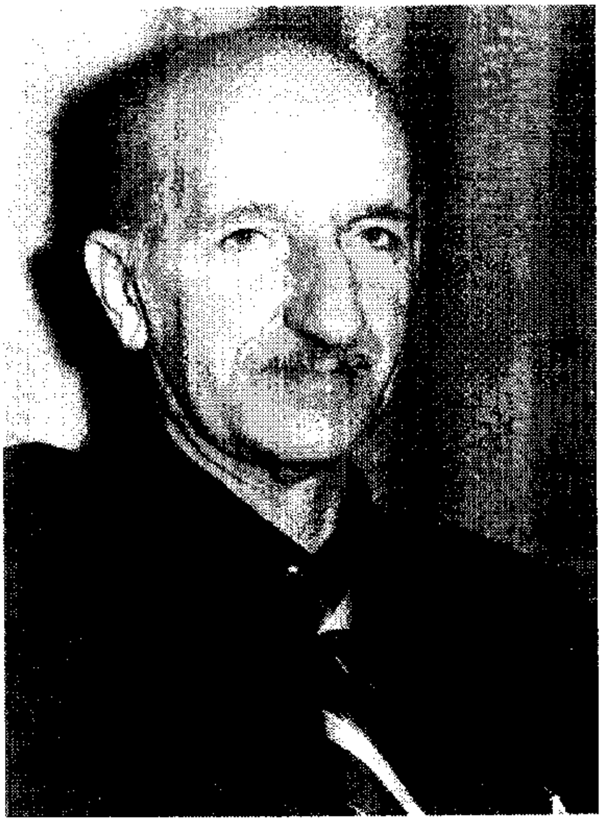

Valentin Danilovich Belousov
(February 20, 1925 - July 23, 1988)
his life and work

This year it has been 80 years since V. D. Belousov's birth. He was born in Beltsy (Bălţi) – a small city located in the north part of the Republic of Moldova where his parents Daniil Afinogenovich Belousov (1897 – 1956) and Elena Konstantinovna Belousova (1897 – 1982) worked at a post office. His father was an officer of the Russian tsarist army. He finished a military school in Tiflis. His ancestral family name was Belous. But together with the diploma of the military school he received a "better" Russian family name Belousov. After the October Revolution he defected from Soviet Union to Moldova which in those days was a part of Romania.

A primary and secondary school V. D. Belousov finished in Beltsy. In the secondary school he was one of the best pupils.

Sometimes during the World War II his father worked in a cash-box at Beltsy oil-factory. In March 1944 the factory and its workers were evacuated to annother part of Romania where Belousov's family lived to November 1944. In that time V. D. Belousov passed examinations to the Bucharest Polytechnical University as one of the best entrances. But he did not study in this university due to his family come back to Beltsy. In December 1944, V. D. Belousov started his study at the Kishinev Pedagogical Institute where lectures of algebra were given by V. A. Andrunakievich who was a pupil of Otto Smidt. In 1947, after graduation Belousov worked for a short time in Kishinev Pedagogical Institute. Next in 1948 – 1950, because of the family reasons, he worked as a teacher in a secondary school in the village Sofia (near Beltsy) where he wrote his first articles on functional equations and quasigroups. The study of quasigroups Belousov began from the reading of Sushkevich's book *The Theory of Generalized Groups* (Harkov–Kiev, 1937). Belousov bought this book on a second-hand market by accident.

In 1950 he began his work at Beltsy Pedagogical Institute and finished it as a chief of the Department of Mathematics. In the period of 1954 – 1955 he participated in a special course of raise of qualifications at Moscow State University where he met A. G. Kurosh who invited Belousov to postgraduate studies (1955 – 1956) and proposed him an investigation of non-associative algebraic structures. The PhD thesis *Studies of quasigroups and loops* was presented in 1958. Many years after V. D. Belousov says that A. G. Kurosh was like his second father.

From October 1960 to July 1961 Belousov was staying at Wisconsin University (Madison, USA), where he worked together with R. H. Bruck.

From 1962 to 1986 he was a chair of the Department of Algebra and Mathematical Logic in the Institute of Mathematics of the Academy of Sciences of the Republic of Moldova. He was also a chair of the Department of Algebra and Geometry (later algebra only) in Kishinev State University (1967 – 1977).

In 1966, at Moscow State University, Belousov presented his doctoral thesis (habilitation) *Systems of quasigroups with generalized identities*. In 1968 he was elected (as a corresponding member) to the Academy of Pedagogical Sciences of the USSR.

V. D. Belousov has made a great contribution to the theory of quasigroups and loops. In the opinion of many famous mathematicians he was the leading specialist in this theory during the sixties and the seventies of the previous century. A. D. Keedwell wrote about him: "*V. D. Belousov, prolific as a researcher, who, like Albert, Bruck and Artzy, engaged in work which was ahead of its time.*"

The full list of Belousov's printed papers is presented below.

Many general questions on quasigroups and loops are studied, for example, in [18], [34], [36], [39] and [83]. Derivative operations of loops are described in [2]; groups of regular mappings – in [4]; nuclei of quasigroups – in [25], [28], [56]; autotopies and antiautotopies – in [84] and [89]. Belousov also characterized groups of inner permutations, normal subquasigroups, isotopy and crossed isotopy [88] and different groups associated with quasigroups [48].

In many papers special classes of quasigroups and loops are investigated: IP-quasigroups [34], F-quasigroups ([12] and [23]), TS-quasigroups ([40] and [77]), CI-quasigroups [46], Stein quasigroups [31], I-quasigroups [83], Bol loops ([28] and [63]), G-loops etc. Belousov's articles contain definitions of new classes of quasigroups and loops, for example, PI-quasigroups, P-quasigroups, S-loops, M-loops, linear quasigroups over groups.

Very important Belousov's results are connected with distributive quasigroups. He proved that every distributive quasigroup is isotopic to a commutative Moufang loop [10]. This fact, together with Toyoda theorem on medial quasigroups, gives a new way to the study of quasigroups: investigations of a class $\mathcal{A}$ of quasigroups can be reduced to the study of a class $\mathcal{B}$ of loops isotopic to quasigroups from $\mathcal{A}$ and properties of this isotopy. Moreover, after this observation, it is clear, that the study of quasigroups "linear" over groups (over loops too) is sensible.

In a series of papers initiated by [3] transitive distributive quasigroups are described. An elegant example of such non-trivial quasigroup is given in [10]. Simultaneously, he found a class of left-distributive quasigroups non-isotopic to groups [17] and characterized connections between some of such quasigroups, Moufang and Bol loops [23]. Loops isotopic to left-distributive quasigroups are studied in [58]. In that paper V. D. Belousov gives several conditions, some of them are necessary and sufficient, some only sufficient, for (1) a loop to be an isotope of a left-distributive quasigroup, (2) a loop isotopic to a left-distributive quasigroup to be a left Bol loop, or a group, or an S-loop, (3) for an S-loop to be a left Bol loop or a commutative Moufang loop, (4) for an S-loop related to a Stein quasigroup to be a left Bol loop. They also investigate some properties of S-loops, such as uniqueness of the relation between a left-distributive quasigroup and an S-loop and the fact that the left and the middle nuclei of an S-loop coincide.

The second series of papers is devoted to different systems of binary operations defined on the same set. Such systems of operations were considered as universal algebras. In particular, Belousov studied systems of quasigroup operations satisfying some laws of distributivity [1], associativity [15], mediality [9] and transitivity [13]. An important results on such systems are contained in [22]. *Positional algebras*, i.e. systems of (full) quasigroup operations (different arities) closed under binary compositions are investigated in [52]. Positional algebras of partial quasigroup operations are investigated in [53]. Now, such systems of operations are called *Belousov algebras* by some authors.

Quasigroups satisfying balanced identities are investigated in [30], [80] and [82]. In [80] it is proved that a quasigroup satisfying a balanced identity in which only one side of this identity contains a subword xy is isotopic to a group. Similar results for quasigroups with completely reducible balanced identities are obtained in [82].

A large cycle of Belousov's articles is connected with different types of functional equations on quasigroups, such as the functional equations of generalized associativity [6], distributivity [24], mediality [67], the functional Moufang equation [51] etc. In solving the functional equation of generalized associativity *Belousov's theorem about four quasigroups* proved in [15] plays a fundamental role. This theorem states that any four quasigroups satisfying the functional equation of associativity are isotopic to the same group. A generalization to the case of quasigroups of different arities is proved in [49]. The case when this equation is not balanced is considered in [57]. The functional equation of mediality is solved in [14] and, in a very general case, in [70]. The functional equation of generalized distributivity is reduced to the simplest form [24] and solved in some special case [41]. The Moufang's functional equation is studied in [47] and [51]. The full solution of the functional balanced equation of the second kind on quasigroups of arbitrary arity is given in [72].

Now balanced equations $w_1 = w_2$ in which for each subsystem u_i of w_i there exists a subsystem v_j of w_j $(i, j = 1, 2)$ with the same set of variables as in u_i are called *Belousov equations*.

Other types of functional equations on quasigroups are considered in [50] and [53]. Some fundamental functional equations on quasigroups on infinitary arity are solved in a series of papers (see [61], [62], [63], [69]) written together with Z. Stojaković.

Belousov had published a number of works devoted to the study of n-ary quasigroups. Now, these works form the foundation of the theory of n-ary quasigroup. As it is well known, there exist n-ary groups (i.e. associative n-

ary quasigroups) without neutral elements and n-ary groups with two, three and more neutral elements. Necessary and sufficient conditions for an n-ary group to have such elements are given in [32]. In that paper is also proved that an n-ary loop isotopic to n-ary group containing neutral element is an n-ary group. Investigations of properties of neutral elements are continued in [33]. The relations between various types of n-ary quasigroups for which its inverse operations are isotopic are described in [85]. Connections between classes of n-ary quasigroups closed under the homotopies are given in [90]. As a consequence, a characterization of some of such classes is obtained by "matrices of homotopies".

Many of Belousov's results on n-ary quasigroups are contained only in his book [59]. This book contains fundamental information on main classes of n-ary quasigroups such as medial quasigroups, TS-quasigroups, Menger quasigroups, Dicker quasigroups, IP-quasigroups, (i,j)-associative quasigroups, positional algebras of quasigroups, reducible quasigroups and some functional equations on these quasigroups.

A natural correspondence between the class of all algebraic 3-nets and the class of all quasigroups are used by Belousov for the study of quasigroups and nets. He start with [35], where one method of obtaining of closure figures is described. Certain transformations in 3-nets preserving some properties of coordatization quasigroup are characterized in [38]. Interrelations between nets and division groupoids are considered in [71]. Closure conditions are studied in [43] and [73] (for k-nets in [54] and [64]). In [74] he proved that every pre-affine plane can be embedded in an affine plane. The problem of coordinatization is discussed in [76], [79] and [86]. These, and other, Belousov's results are presented in his book [55]. A survey [60] contains the main results obtained by him and other authors in the period of 1954 – 1971.

Quasigroups have many applications in discrete mathematics, especially in the theory of Latin squares [87], because every finite quasigroup has a Latin square as its Cayley table and, conversely, every Latin square is the multiplication table of a certain quasigroup. A similar connection hold between n-ary quasigroups and n-dimensional cubes. In connection with this fact V. D. Belousov studied the problem of extensions of quasigroups [39] and systems of orthogonal binary [42] and n-ary operations [68]. In [42] he established the connection between orthogonal systems of operations and orthogonal systems of quasigroups (OSQ) and studied the parastrophy transformation of this OSQ. In this study he used the fact that a pair of binary operations (Q, A) and (Q, B) defines a map θ of the set Q^2 such that $\theta(x,y) = (A(x,y), B(x,y))$. It is clear that the operations A and B are orthogonal if and only if θ is a permutation of the set Q^2. In opinion of many specialists this approach is one of the most general and effective approaches to the orthogonality of binary operations.

The first description of minimal identities connected with the orthogonality of parastrophes of binary quasigroups is given in [81]. Some interrelations between the orthogonality of quasigroups and closure operations in k-nets are proved in [54].

V. D. Belousov was the first one to begin regular study of quasigroups linear over groups. Also at first in a large scope he used isotopies to the translation of many problems on quasigroups to the corresponding problems on loops and reduced investigations of these problems for the investigation of the corresponding problems on loops and the properties of this isotopy.

All ideas mentioned above are now in development by numerous Belousov's pupils and by many other mathematicians. For about 30 of them he was a supervisor of their Ph. D. thesis. Now these pupils work in many countries.

V. D. Belousov was a member of the editorial boards of *Aequationes Mathematicae* and *Buletinul* (*Izviestya*) *of the Academy of Sciences of the Republic Moldova*, ser. *Matematica*. He was an editor of the known series of collected works *Matematicheskiye Issledovaniya* (*Mathematical Researches*) printed in Moldova.

He was not only a famous scientist. He was a very good lecturer. As the Corresponding Member of the Academy of the Pedagogical Sciences of the USSR (section Mathematics) he was an organizer of the scientific life in Moldova. For a very long time he was a chairman of the jury of the Moldavian mathematical olympiad for schoolboys. He was a co-author of the book *Republican mathematical olympiad* (Ştiinţa, Kishinev 1986) and the *Russian-Moldavian mathematical dictionary* (Kishivev 1980) which contains 24 000 items.

For many years he had been a deputy of the parliament of the Republic of Moldova and the city council of Beltsy.

As a person, V. D. Belousov was full of generosity and warmth. Communications with him was very pleasant. Belousov wrote epigrams, liked music, especially Moldavian folk music and music of V. A. Mozart. He had a large library (in many languages) of science and fiction literature.He was also a vine and cognac connoisseur.

Belousov once says that he is not known because one student on an algebraic seminar for students (when Belousov was a lecturer of the Kishinev University) translated the term "the Belousov equation" as "an equation with a white beard" (in Russian "belousov" can be translated to "a man with white beard").

His wife, Elizaveta Fedorovna (1925 – 1991), was a teacher of Russian language and Russian literature at Moldova State University for a long time. The son Alexander (1948 – 1998), Doctor of Physics, was a promising physicist-theorist. The daughter Tatiana Valentinovna Kravtchenko, is a medical doctor (neuropathologist) and mother of two children.

Grandsons of V. D. Belousov, Natalia Alexandrovna Belousova and Petr Kravtchenko both finished Moldova State University, Faculty of Mathematics and Cybernetics. They are mathematicians. Natalia lives in Holland, Petr in Kishinev. The brother of V. D. Belousov, Victor (1927 – 2004) was a professor of medicine. The son of Victor, Igor Victorovich Belousov is a professor of physics. That way, like V. D. Belousov said, the wish of his father that his sons Valentin and Victor would become professors has been fulfilled.

Valentin Danilovich Belousov devoted his life to science, a life that will always be an example and inspiration to his followers.

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