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Anti fuzzy Lie ideals of Lie algebras

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Abstract

In this paper we apply the Biswas's idea of anti fuzzy subgroups to Lie ideals of Lie algebras. We introduce the notion of anti fuzzy ideals in Lie algebras and investigate some of their properties.

1. Introduction

Lie algebras were discovered by Sophus Lie (1842-1899) while he was attempting to classify certain "smooth" subgroups of general linear groups. The groups he considered are now called Lie groups. He found that by taking the tangent space at the identity element of such a group, one obtained a Lie algebra. Problems about the group could be reduced to problems about the Lie algebra in which form they usually proved more tractable. There are many applications of Lie algebras, such as spectroscopy of molecules, atoms, nuclei and hadrons. Physical applications of Lie algebras include rotations and vibrations of molecules (vibron model), collective modes in nuclei (interacting boson model), the atomic shell model, the nuclear shell model, and the quark model of hadrons.

The notion of fuzzy sets was first introduced by L. A. Zadeh [12]. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics. There have been wide-ranging applications of the theory of fuzzy sets, from the design of robots and computer simulation to engineering and water resources planning. A. Rosenfeld [9] introduced the fuzzy sets in the realm of group theory. Since then many mathematicians have been involved in extending the concepts and results of abstract algebra to the

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broader frame work of the fuzzy setting. Fuzzy ideals in Lie algebras have been studied in [2, 3, 7, 8, 10, 11]. In this paper we apply the Biswas's idea of anti fuzzy subgroups to Lie ideals of Lie algebras. We introduce the notion of anti fuzzy ideals in Lie algebras and investigate some of their properties.

2. Preliminaries

In this section we review some elementary aspects that are necessary for this paper.

Definition 2.1. A *Lie algebra* is a vector space L over a field F (equal to $\mathbf{R}$ or $\mathbf{C}$) on which $L \times L \to L \quad (x, y) \to [x, y]$ is defined satisfying the following axioms:

(L1) $[x, y]$ is bilinear,

(L2) $[x, x] = 0$ for all $x \in L$,

(L3) $[[x, y], z] + [[y, z], x] + [[z, x], y] = 0$ for all $x, y, z \in L$ (Jacobi identity).

In this paper by L will be denoted a Lie algebra. We note that the multiplication in a Lie algebra is not associative, i.e., it is not true in general that $[[x, y], z] = [x, [y, z]]$. But it is *anti commutative*, i.e., $[x, y] = -[y, x]$.

Definition 2.2. Let L_1 and L_2 be Lie algebras over a field F. A linear transformation $f : L_1 \to L_2$ is called a *Lie homomorphism* if $f([x, y]) = [f(x), f(y)]$ for all $x,\ y \in L_1$.

Definition 2.3. A subspace H of Lie algebra L is called *Lie subalgebra* if $[x, y] \in H$ for $x, y \in H$. A subspace I of L is called *Lie ideal* of Lie algebra if for all $x \in I,\ y \in L$ implies $[x, y] \in I$, i.e., $[I, L] \subseteq I$.

Definition 2.4. A *fuzzy set* γ, i.e., a map $\gamma : L \to [0, 1]$, is called a *fuzzy Lie subalgebra* of L if

(a) $\gamma(x + y) \geqslant \min\{\gamma(x), \gamma(y)\}$,

(b) $\gamma(\alpha x) \geqslant \gamma(x)$,

(c) $\gamma([x, y]) \geqslant \min\{\gamma(x), \gamma(y)\}$

hold for all $x, y \in L$ and $\alpha \in F$.

Definition 2.5. A fuzzy subset $\gamma : L \to [0, 1]$ satisfying (a), (b) and

(d) $\gamma([x, y]) \geqslant \gamma(x)$

is called a *fuzzy Lie ideal* of L.

A fuzzy ideal of L is a fuzzy subalgebra [2] such that $\gamma(-x) \geqslant \gamma(x)$ holds for all $x \in L$. According to Zadeh's extension principle the bracket $[\cdot, \cdot]$ on L can be extended to the bracket $\ll \cdot, \cdot \gg$ defined on the set of all anti fuzzy sets on L in the following way

$$\ll \gamma, \lambda \gg (x) = \inf\{\max\{\gamma(y), \lambda(z)\} \,|\, y, z \in L,\, [y, z] = x\},$$

where γ, λ are anti fuzzy sets on L and $x \in L$.

3. Anti fuzzy Lie ideals

Definition 3.1. Let L be a Lie algebra. A fuzzy subset γ of L is called an *anti fuzzy Lie ideal* of L if the following axioms are satisfied:

(AF1) $\gamma(x + y) \leqslant \max(\{\gamma(x), \gamma(y)\}$,

(AF2) $\gamma(\alpha x) \leqslant \gamma(x)$,

(AF3) $\gamma([x, y]) \leqslant \gamma(x)$ for all $x, y \in L$ and $\alpha \in F$.

Example 3.2. Let $\Re^2 = \{(x, y) : x, y \in R\}$ be the set of all 2-dimensional real vectors. Then $\Re^2$ with $[x, y] = x \times y$ is a real Lie algebra. Define a fuzzy set of $\Re^2$ by

$$\gamma(x, y) = \begin{cases} 0 & \text{if } x = y = 0, \\ 1 & \text{otherwise.} \end{cases}$$

By routine computations, we can easily check that γ is an anti fuzzy Lie ideal of $\Re^2$.

The following lemma is obvious.

Lemma 3.3. *Let γ be an anti fuzzy Lie ideal of L then*

(i) $\gamma(0) \leqslant \gamma(x) \quad \forall\, x \in L$,

(ii) $\gamma([x, y]) \leqslant \min\{\gamma(x), \gamma(y)\} \ \ \forall\, x, y \in L$,

(iii) $\gamma([x, y]) = \gamma(-[y, x]) = \gamma([y, x]) \ \ \forall x, y \in L$.

Theorem 3.4. *Let γ be an anti fuzzy Lie ideal in a Lie algebra L. Then γ is an anti fuzzy Lie ideal of L if and only if the set $L(\gamma;t) = \{x \in L|\gamma(x) \leqslant t\}$, $t \in [0,1]$, is a Lie ideal of L when it is nonempty.*

Proof. Assume that γ is an anti fuzzy Lie ideal of L and let $t \in [0,1]$ be such that $L(\gamma;t) \neq \emptyset$. Let x, $y \in L$ be such that $x \in L(\gamma;t)$, and $y \in L(\gamma;t)$. Then $\gamma(x) \leqslant t$ and $\gamma(y) \leqslant t$. It follows that

$$\gamma(x+y) \leqslant \max\{\gamma(x), \gamma(y)\} \leqslant t,$$

$$\gamma(\alpha x) \leqslant \gamma(x) \leqslant t,$$

$$\gamma([x,y]) \leqslant \gamma(x) \leqslant t$$

so that $x + y \in L(\gamma;t)$, $\alpha x \in L(\gamma;t)$ and $[x,y] \in L(\gamma;t)$. Hence $L(\gamma;t)$ is a Lie ideal of L.

Conversely, suppose that $L(\gamma;t) \neq \emptyset$ is a Lie ideal of L for every $t \in [0,1]$. Assume that $\gamma(x+y) > \max\{\gamma(x), \gamma(y)\}$ for some $x, y \in L$. Taking

$$t_0 := \frac{1}{2}\{\gamma(x+y) + \max\{\gamma(x) + \gamma(y)\}\},$$

we have $\gamma(x+y) > t_0 > \max\{\gamma(x), \gamma(y)\}$. So, $x + y \notin L(\gamma;t)$, $x \in L(\gamma;t)$ and $y \in L(\gamma;t)$. This is a contradiction. Hence $\gamma(x+y) \leqslant \max\{\gamma(x), \gamma(y)\}$ for all $x, y \in L$.

Similarly we can show that $\gamma(\alpha x) \leqslant \gamma(x)$ and $\gamma([x,y]) \leqslant \gamma(x)$. This completes the proof. □

Theorem 3.5. *If γ and ρ are anti fuzzy Lie ideals of a Lie algebra L, then the function $\gamma \vee \rho : L \to [0,1]$ defined by*

$$(\gamma \vee \rho)(x) = \max\{\gamma(x), \rho(x)\}$$

is an anti fuzzy Lie ideal of L.

Proof. Let $x, y \in L$ and $\alpha \in F$. Then

$$\begin{aligned}(\gamma \vee \rho)(x+y) &= \max\{\gamma(x+y), \rho(x+y)\}\\ &\leq \max\{\max\{\gamma(x), \gamma(y)\}, \max\{\rho(x), \rho(y)\}\}\\ &= \max\{\max\{\gamma(x), \rho(x)\}, \max\{\gamma(y), \rho(y)\}\}\\ &= \max\{(\gamma \vee \rho)(x), (\gamma \vee \rho)(y)\},\end{aligned}$$

$$(\gamma \vee \rho)(ax) = \max\{\gamma(ax), \rho(ax)\} \leqslant \max\{\gamma(x), \rho(x)\} = (\gamma \vee \rho)(x),$$

$$(\gamma \vee \rho)([x,y]) = \max\{\gamma([x,y]), \rho([x,y])\} \leqslant \max\{\gamma(x), \rho(x)\} = (\gamma \vee \rho)(x).$$

Hence $(\gamma \vee \rho)$ is an anti fuzzy Lie ideal of L. □

Definition 3.6. For a family of fuzzy sets $\{\gamma_i | i \in I\}$ in a Lie algebra L, the *union* $\bigvee \gamma_i$ of $\{\gamma_i | i \in I\}$ is defined by

$$(\bigvee \gamma_i)(x) = \sup\{\gamma_i(x) | i \in I\},$$

for each $x \in L$.

Theorem 3.7. *If $\{\gamma_i | i \in I\}$ is a family of anti fuzzy Lie ideals of Lie algebras L then so is $\bigvee \gamma_i$.*

Proof. Straightforward. □

Theorem 3.8. *Let $f : L_1 \to L_2$ be an epimorphism of Lie algebras. If ν is an anti fuzzy Lie ideal of L_2 and γ is the pre-image of ν under f. Then γ is an anti fuzzy Lie ideal of L_1.*

Proof. For any $x, y \in L_1$ and $\alpha \in F$,

$$\begin{aligned}\gamma(x+y) &= \nu(f(x+y)) = \nu(f(x)+f(y)) \\ &\leq \max\{\nu(f(x)), \nu(f(y))\} = \max\{\gamma(x), \gamma(y)\}, \\ \gamma(\alpha x) &= \nu(f(\alpha x)) = \nu(\alpha f(x)) \leq \nu(f(x)) = \gamma(x),\end{aligned}$$

and

$$\gamma([x,y]) = \nu(f([x,y])) \leq \nu(f(x)) = \gamma(x).$$

Hence γ is an anti fuzzy Lie ideal of L_1. □

Definition 3.9. Let L_1 and L_2 be two Lie algebras and f be a function of L_1 into L_2. If γ is a fuzzy set in L_2, then the *preimage* of γ under f is the fuzzy set in L_1 defined by

$$f^{-1}(\gamma)(x) = \gamma(f(x)) \quad \forall x \in L_1.$$

Theorem 3.10. *Let $f : L_1 \to L_2$ be an onto homomorphism of Lie algebras. If γ is an anti fuzzy Lie ideal of L_2, then $f^{-1}(\gamma)$ is an anti fuzzy Lie ideal of L_1.*

Proof. Let $x_1, x_2 \in L_1$ and $\alpha \in F$, then

$$\begin{aligned}f^{-1}(\gamma)(x_1+x_2) &= \gamma(f(x_1)+f(x_2)) \leqslant \max\{\gamma(f(x_1)), \gamma(f(x_2))\} \\ &= \max\{f^{-1}(\gamma)(x_1), f^{-1}(\gamma)(x_2)\}, \\ f^{-1}(\gamma)(\alpha x_1) &= \gamma(f(\alpha x_1)) \leqslant \gamma(\alpha f(x_1)) = \alpha f^{-1}(\gamma)(x_1), \\ f^{-1}(\gamma)([x,y]) &= \gamma(f([x,y])) = \gamma([f(x), f(y)]) \leqslant \gamma(f(x)) = f^{-1}(\gamma)(x).\end{aligned}$$

Hence $f^{-1}(\gamma)$ is an anti fuzzy Lie ideal of L_1. □

Theorem 3.11. *Let $f : L_1 \to L_2$ be an onto homomorphism of Lie algebras. If γ is an anti fuzzy Lie ideal of L_2, then $f^{-1}(\gamma^c) = (f^{-1}(\gamma))^c$.*

Proof. Let γ be an anti fuzzy Lie ideal of L_2. Then for $x \in L_1$,

$$f^{-1}(\gamma^c)(x) = \gamma^c(f(x)) = 1 - \gamma(f(x)) = 1 - f^{-1}(\gamma^c)(x) = (f^{-1}(\gamma))^c(x).$$

That is $f^{-1}(\gamma^c) = (f^{-1}(\gamma))^c$. □

Definition 3.12. Let γ be a fuzzy set in a Lie algebra L and f a mapping defined on L. Then the fuzzy set γ^f in $f(L)$ defined by

$$\gamma^f(y) = \inf_{x \in f^{-1}(y)} \gamma(x)$$

for every $y \in f(L)$, is called the *image* of γ under f. A fuzzy set γ in L has the inf *property* if for any subset $A \subseteq L$, there exists $a_0 \in A$ such that $\gamma(a_0) = \inf\limits_{a \in A} \gamma(a)$.

Theorem 3.13. *A Lie algebra homomorphism image of an anti fuzzy Lie ideal having the* inf *property is an anti fuzzy Lie ideal.*

Proof. Let $f : L_1 \to L_2$ be an epimorphism of L_1 onto L_2 and γ be a fuzzy Lie ideal of L_1 with the inf property. Consider $f(x), f(y) \in f(L_1)$. Let $x_0, y_0 \in f^{-1}(f(x))$ be such that

$$\gamma(x_0) = \inf_{t \in f^{-1}(f(x))} \gamma(t) \quad \text{and} \quad \gamma(y_0) = \inf_{t \in f^{-1}(f(y))} \gamma(t)$$

respectively. Then

$$\begin{aligned} \nu(f(x) + f(y)) &= \inf_{t \in f^{-1}(f(x)+f(y))} \gamma(t) \leqslant \gamma(x_0 + y_0) \leqslant \max\{\gamma(x_0) + \gamma(y_0)\} \\ &= \max\{\inf_{t \in f^{-1}(f(x))} \gamma(t), \inf_{t \in f^{-1}(f(y))} \gamma(t)\} \\ &= \max\{\nu(f(x)) + \nu(f(y))\}, \end{aligned}$$

$$\nu(f(\alpha x)) = \inf_{t \in f^{-1}(f(\alpha x))} \gamma(t) \leqslant \gamma(x_0) \leqslant \max\{\gamma(x_0)\} = \nu(f(x)),$$

$$\begin{aligned} \nu([f(x), f(y)]) = \nu(f([x, y])) &= \inf_{t \in f^{-1}(f([x,y]))} \gamma(t) \leqslant \gamma([x_0, y_0]) \\ &\leqslant \gamma(x_0) = \nu(f(x)). \end{aligned}$$

Consequently, ν is an anti fuzzy Lie ideal of L_2. □

Definition 3.14. Let L_1 and L_2 Lie algebras and f a function of γ is a fuzzy set in L_1, then the *anti image* of γ under f is the fuzzy set defined by $f(\gamma)(y) =$

$$\begin{cases} \inf\{\gamma(t) \mid t \in L_1, f(t) = y\}, & \text{if } f^{-1}(y) \neq \emptyset \ , \\ 1, & \text{otherwise.} \end{cases}$$

Definition 3.15. Let L_1 and L_2 be any sets and let $f : L_1 \to L_2$ be any function. A fuzzy set γ is called *f-invariant* if and only if for $x, y \in L_1$, $f(x) = f(y)$ implies $\gamma(x) = \gamma(y)$.

Theorem 3.16. *Let $f : L_1 \to L_2$ be an epimorphism of Lie algebras. Then γ is an f-invariant anti fuzzy Lie ideal of L_1 if and only if $f(\gamma)$ is an anti fuzzy Lie ideal of L_2.*

Proof. Let $x, y \in L_2$ and $\alpha \in F$. Then there exist $a, b \in L_1$ such that $f(a) = x$, $f(b) = y$, $x + y = f(a + b)$ and $\alpha x = \alpha f(a)$. Since γ is f-invariant,

$$\begin{aligned} f(\gamma)(x+y) &= \gamma(a+b) \leqslant \max\{\gamma(a), \gamma(b)\} = \max\{f(\gamma)(x), f(\gamma)(y)\}, \\ f(\gamma)(\alpha x) &= \gamma(\alpha a) \leqslant \gamma(a) = f(\gamma)(x), \\ f(\gamma)([x,y]) &= \gamma([a,b]) = [\gamma(a), \gamma(b)] \leqslant \gamma(a) = f(\gamma)(x). \end{aligned}$$

Hence $f(\gamma)$ is an anti fuzzy Lie ideal of L_2.

Conversely, if $f(\gamma)$ is an anti fuzzy Lie ideal of L_2, then for any $x \in L_1$

$$\begin{aligned} f^{-1}(f(\gamma))(x) &= f(\gamma)(f(x)) = \inf\{\gamma(t) \mid t \in L_1, f(t) = f(x)\} \\ &= \inf\{\gamma(t) \mid t \in L_1, \gamma(t) = \gamma(x)\} = \gamma(x). \end{aligned}$$

Hence $f^{-1}(f(\gamma)) = \gamma$ is an anti fuzzy Lie ideal by Theorem 3.10. □

Definition 3.17. An ideal A of Lie algebra L is said to be *characteristic* if $f(A) = A$, for all $f \in \mathrm{Aut}(L)$, where $\mathrm{Aut}(L)$ is the set of all automorphisms of L. Anti fuzzy Lie ideal γ of Lie algebra L is said to be *anti fuzzy characteristic* if $\gamma^f(x) = \gamma(x)$, for all $x \in L$ and $f \in\mathrm{Aut}(L)$.

Lemma 3.18. *Let γ be an anti fuzzy Lie ideal of a Lie algebra L and let $x \in L$. Then $\gamma(x) = s$ if and only if $x \in L(\gamma; s)$ and $x \notin L(\gamma; t)$, for all $s > t$.*

Proof. Straightforward. □

Theorem 3.19. *An anti fuzzy Lie ideal is characteristic if and only if each its level set is a characteristic Lie ideal.*

Proof. Suppose that γ is anti fuzzy characteristic and let $s \in Im(\gamma)$, $f \in$Aut(L) and $x \in L(\gamma; s)$. Then $\gamma^f(x) = \gamma(x)$ implies $\gamma(f(x)) \leqslant s$ whence $f(x) \in L(\gamma; s)$. Thus $f(L(\gamma; s)) \subseteq L(\gamma; s)$.

Let $x \in L(\gamma; s)$ and $y \in L$ such that $f(y) = x$. Then $\gamma(y) = \gamma^f(y) = \gamma(f(y)) = \gamma(x) \leqslant s$, consequently $y \in L(\gamma; s)$. So, $x = f(y) \in L(\gamma; s)$. Thus, $L(\gamma; s) \subseteq f(L(\gamma; s))$. Hence $f(L(\gamma; s)) = L(\gamma; s)$, i.e., $L(\gamma; s)$ is characteristic.

Conversely, suppose that each level Lie ideal of γ is characteristic and let $x \in L$, $f \in Aut(L)$, $\gamma(x) = s$. Then, by virtue of Lemma 3.18, $x \in L(\gamma; s)$ and $x \notin L(\gamma; t)$, for all $s > t$. It follows from the assumption that $f(x) \in f(L(\gamma; s)) = L(\gamma; s)$, so that $\gamma^f(x) = \gamma(f(x))) \leqslant s$. Let $t = \gamma^f(x)$ and assume that $s > t$. Then $f(x) \in L(\gamma; t) = f(L(\gamma; t))$, which implies from the injectivity of f that $x \in L(\gamma; t)$, a contradiction. Hence $\gamma^f(x) = \gamma(f(x)) = s = \gamma(x)$ showing that γ is an anti fuzzy characteristic. □

Definition 3.20. Let γ be an anti fuzzy Lie ideal in L. Define a sequence of anti fuzzy Lie ideals in L putting $\gamma^0 = \gamma$ and $\gamma^n = [\gamma^{n-1}, \gamma^{n-1}]$ for $n > 0$. If there exists a positive integer n such that $\gamma^n = 0$, then an anti fuzzy Lie ideal γ is called *solvable.*

Theorem 3.21. *Homomorphic image of a solvable anti fuzzy Lie ideal is a solvable anti fuzzy Lie ideal.*

Proof. Let $f : L_1 \to L_2$ be a homomorphism of Lie algebras. Suppose that γ is a solvable anti fuzzy Lie ideal in L_1. We prove by induction on n that $f(\gamma^n) \supseteq [f(\gamma)]^n$, where n is any positive integer. First we claim that $f([\gamma, \gamma]) \supseteq [f(\gamma), f(\gamma)]$. Let $y \in L_2$, then

$$\begin{aligned}
&f(\ll \gamma, \gamma \gg)(y) = \inf\{\ll \gamma, \gamma \gg (x) \mid f(x) = y\}\\
&= \inf\{\inf\{\max\{\gamma(a), \gamma(b)\} \mid a, b \in L_1, [a, b] = x, f(x) = y\}\}\\
&= \inf\{\max\{\gamma(a), \gamma(b)\} \mid a, b \in L_1, [a, b] = x, f(x) = y\}\\
&= \inf\{\max\{\gamma(a), \gamma(b)\} \mid a, b \in L_1, [f(a), f(b)] = x\}\\
&= \inf\{\max\{\gamma(a), \gamma(b)\} \mid a, b \in L_1, f(a) = u, f(b)] = v, [u, v] = y\}\\
&\leq \inf\{\max\{\inf_{a\in f^{-1}(u)} \gamma(a), \inf_{b\in f^{-1}(v)} \gamma(b)\} \mid [u, v] = y\}\\
&= \inf\{\max(f(\gamma)(u), f(\gamma)(v)) \mid [u, v] = y\} =\ll f(\gamma), f(\gamma) \gg (y).
\end{aligned}$$

Now for $n > 1$, we get $f(\gamma^n) = f([\gamma^{n-1}, \gamma^{n-1}]) \supseteq [f(\gamma^{n-1}), f(\gamma^{n-1})] \supseteq [(f(\gamma))^{n-1}, (f(\gamma))^{n-1}] = (f(\gamma))^n$. This completes the proof. □

Definition 3.22. Let γ be an anti fuzzy Lie ideal in L and let $\gamma_n = [\gamma, \gamma_{n-1}]$ for $n > 0$, where $\gamma_0 = \gamma$. If there exists a positive integer n such that $\gamma_n = 0$ then γ is called *nilpotent*.

Using the same method as in the proof of Theorem 3.21, we can prove the following two theorems.

Theorem 3.23. *Homomorphic image of a nilpotent anti fuzzy Lie ideal is a nilpotent anti fuzzy Lie ideal.*

Theorem 3.24. *If γ is a nilpotent anti fuzzy Lie ideal, then it is solvable.*

Theorem 3.25. *Let I be a Lie ideal of a Lie algebra L. If γ is an anti fuzzy Lie ideal of L, then the fuzzy set $\overline{\gamma}$ of L/I defined by*

$$\overline{\gamma}(a+I) = \inf_{x\in I} \gamma(a+x)$$

is an anti fuzzy Lie ideal of the quotient Lie algebra L/I.

Proof. Clearly, $\overline{\gamma}$ is well-defined. Let $x+I,\ y+I \in L/I$, then

$$\begin{aligned}
\overline{\gamma}(x+I)+(y+I)) &= \overline{\gamma}_A((x+y)+I) = \inf_{z\in I} \gamma((x+y)+z) \\
&= \inf_{z=s+t\in I} \gamma((x+y)+(s+t)) \\
&\leqslant \inf_{s,\ t\in I} \max\{\gamma(x+s), \gamma(y+t)\} \\
&= \max\{\inf_{s\in I} \gamma(x+s), \inf_{t\in I} \gamma(y+t)\} \\
&= \max\{\overline{\gamma}(x+I), \overline{\gamma}(y+I)\},
\end{aligned}$$

$$\overline{\gamma}(\alpha(x+I)) = \overline{\gamma}(\alpha x+I) = \inf_{z\in I} \gamma(\alpha x+z) \leqslant \inf_{z\in I} \gamma(x+z) = \overline{\gamma}(x+I),$$

$$\overline{\gamma}([x+I, y+I]) = \overline{\gamma}([x,y]+I) = \inf_{z\in I} \gamma([x,y]+z) \leqslant \inf_{z\in I} \gamma(x+z) = \overline{\gamma}(x+I).$$

Hence $\overline{\gamma}$ is an anti fuzzy Lie ideal of L/I. □

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