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On prolongations of quasigroups

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Abstract

We prove that any quasigroup admissing complete or quasicomplete mapping has a prolongation to a quasigroup having one element more.

1. Introduction

By a *prolongation* of a quasigroup we mean a process which shows how, starting from a quasigroup $Q(\cdot)$ of order n, we can obtain a quasigroup $Q'(\circ)$ of order $n+1$ such that the set Q' is obtained from the set Q by the adjunction of one additional element. In other words, it is a process which shows how a given Latin square extends to a new Latin square by the adjunction of one additional row and one column. The first construction of prolongation was proposed by R. H. Bruck [7] who considered only the case of idempotent quasigroups. More general construction was given by J. Dénes and K. Pásztor [9]. Further generalizations, for special types of quasigroups, have been discussed in [2] and [3] by V. D. Belousov. In fact, the construction proposed by V. D. Belousov is more elegant form of the construction proposed by J. Dénes and K. Pásztor. G. B. Belyavskaya studied this problem together with the inverse problem, i.e., with the problem how from a given Latin square of order n one can obtain a Latin square of order $n-1$ (cf. [4, 5, 6]). Quasigroups obtained by the construction proposed by G. B. Belyavskaya are not isotopic to quasigroups obtained by the constructions proposed by R. H. Bruck and V.D. Belousov. This means that we have two different methods of construction of prolongations.

Below we present a third method. Our method can be applied to any quasigroup of order n with the property that its multiplication table possesses a partial transversal of length $n-1$, i.e., a sequence of $n-1$ distinct elements contained in distinct rows and distinct columns. All these three constructions are presented in short elegant form.

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2. Definitions and basic facts

In this paper $Q(\cdot)$ always denotes a quasigroup. The set Q' is identified with the set $Q \cup \{q\}$, where $q \notin Q$.

Any mapping σ of a quasigroup $Q(\cdot)$ defines on Q a new mapping $\overline{\sigma}$, called *conjugated to* σ, such that

$$\overline{\sigma}(x) = x \cdot \sigma(x) \tag{1}$$

for all $x \in Q$. If σ is the identity mapping ε, then $\overline{\sigma}(x) = x^2$. The set

$$\operatorname{def}(\sigma) = Q \setminus \overline{\sigma}(Q),$$

where $\overline{\sigma}(Q) = \{\overline{\sigma}(x) \,|\, x \in Q\}$, is called the *defect* of σ.

A mapping σ is *quasicomplete* on a quasigroup $Q(\cdot)$ if $\overline{\sigma}(Q)$ contains all elements of Q except one. In this case there exists an element $a \in Q$, called *special*, such that $a = \overline{\sigma}(x_1) = \overline{\sigma}(x_2)$ for some $x_1, x_2 \in Q$. If $\overline{\sigma}(Q)$ contains all elements of Q, then we say that σ is *complete*. A quasigroup having at least one complete mapping is called *admissible*. V. D. Belousov proved in [3] (see also [2]) that any admissible quasigroup is isotopic to some idempotent quasigroup and has a prolongation. Since for a given admissible quasigroup the method of constructions of a prolongation proposed by V.D. Belousov gives, in fact, a quasigroup which is isotopic to a quasigroup obtained from the corresponding idempotent quasigroup (by the method proposed by R. H. Bruck) we will identify these two methods and will call it the *classical construction*.

3. Prolongations of admissible quasigroups

1. Classical constructions. The idea of the construction proposed by R. H. Bruck is presented by the following tables, where the corresponding empty cells of these tables are identical.

$$\begin{array}{c|cccccc|}
\cdot & 1 & 2 & 3 & 4 & \dots & n \\ \hline
1 & 1 & & & & & \\
2 & & 2 & & & & \\
3 & & & 3 & & & \\
4 & & & & 4 & & \\
\vdots & & & & & \ddots & \\
n & & & & & & n \\ \hline
\end{array}
\quad\longrightarrow\quad
\begin{array}{c|cccccc|c}
\circ & 1 & 2 & 3 & 4 & \dots & n & q \\ \hline
1 & q & & & & & & 1 \\
2 & & q & & & & & 2 \\
3 & & & q & & & & 3 \\
4 & & & & q & & & 4 \\
\vdots & & & & & \ddots & & \vdots \\
n & & & & & & q & n \\ \hline
q & 1 & 2 & 3 & 4 & \dots & n & q
\end{array}$$

The quasigroup $Q'(\circ)$ obtained from the quasigroup $Q(\cdot)$ is a loop with the identity q. The operation on Q' is defined according to the formula:

$$x \circ y = \begin{cases} x \cdot y & \text{for} \quad x, y \in Q,\ x \neq y, \\ x & \text{for} \quad x \in Q,\ y = q, \\ y & \text{for} \quad x = q,\ y \in Q, \\ q & \text{for} \quad x = y \in Q'. \end{cases} \tag{2}$$

In the construction for a prolongation of an admissible quasigroup $Q(\cdot)$ proposed by V. D. Belousov [3] the complete mapping σ of $Q(\cdot)$ and its conjugated mapping $\overline{\sigma}$ are used. The operation on Q' is defined by the formula:

$$x \circ y = \begin{cases} x \cdot y & \text{for} \quad x, y \in Q,\ y \neq \sigma(x), \\ \overline{\sigma}(x) & \text{for} \quad x \in Q,\ y = q, \\ \overline{\sigma}\sigma^{-1}(y) & \text{for} \quad x = q,\ y \in Q, \\ q & \text{for} \quad x \in Q,\ y = \sigma(x), \\ q & \text{for} \quad x = y = q. \end{cases} \tag{3}$$

Geometrically this means that the multiplication table (Latin square) $L' = [a'_{ij}]$ of a quasigroup $Q'(\circ)$ is obtained from the multiplication table $L = [a_{ij}]$ of a quasigroup $Q(\cdot)$ by the adjunction of one row and one column in this way that all elements from the cells $a_{i\sigma(i)}$ are moved to the last place of the i-th row and $\sigma(i)$-th column of L'. Elements of the cells $a_{i\sigma(i)}$ are replaced by $q = n + 1$. Additionally we put $a_{qq} = q$. In other words: $a'_{ij} = a_{ij}$ for $i \neq \sigma(i)$, $a'_{iq} = a_{i\sigma(i)} = \overline{\sigma}(i)$, $a'_{qj} = a_{\sigma^{-1}(j)j} = \overline{\sigma}\sigma^{-1}(j)$ and $a'_{i\sigma(i)} = a'_{qq} = q$.

Example 1. Consider the quasigroup $Q(\cdot)$ with the multiplication table

$\cdot$	1	2	3	4	5
1	1	2	3	4	5
2	4	3	1	5	2
3	2	5	4	1	3
4	5	4	2	3	1
5	3	1	5	2	4

and its two complete mappings $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 4\,2\,1\,5\,3 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 3\,1\,2\,5\,4 \end{pmatrix}$.
Then, as it is not difficult to see, $\overline{\sigma} = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 4\,3\,2\,1\,5 \end{pmatrix}$ and $\overline{\tau} = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 3\,4\,5\,1\,2 \end{pmatrix}$.

Using these two mappings we can construct two different prolongations:

$\circ_1$	1	2	3	4	5	6
1	1	2	3	6	5	4
2	4	6	1	5	2	3
3	6	5	4	1	3	2
4	5	4	2	3	6	1
5	3	1	6	2	4	5
6	2	3	5	4	1	6

$\circ_2$	1	2	3	4	5	6
1	1	2	6	4	5	3
2	6	3	1	5	2	4
3	2	6	4	1	3	5
4	5	4	2	3	6	1
5	3	1	5	6	4	2
6	4	5	3	2	1	6

The first prolongation is obtained by σ, the second by τ.

By transpositions of rows and columns, we can transform these two tables into multiplication tables of loops $Q'(\star_1)$ and $Q'(\star_2)$:

$\star_1$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	5	6	1	4
3	3	1	6	5	4	2
4	4	6	1	3	2	5
5	5	4	2	1	6	3
6	6	5	4	2	5	1

$\star_2$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	6	5	1	3	4
3	3	1	2	6	4	5
4	4	5	6	2	1	3
5	5	4	1	3	6	2
6	6	3	4	5	2	1

Since $\gamma(x \star_1 y) = \alpha(x) \star_2 \beta(y)$, where

$$\alpha = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 2\,4\,1\,6\,5\,3 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 4\,1\,6\,3\,5\,2 \end{pmatrix}, \qquad \gamma = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 1\,2\,4\,5\,3\,6 \end{pmatrix},$$

loops $Q'(\star_1)$ and $Q'(\star_2)$ are isotopic. This means that also prolongations $Q'(\circ_1)$ and $Q'(\circ_2)$ are isotopic. $\square$

If the diagonal of the multiplication table of a quasigroup $Q(\cdot)$ contains all elements of Q, then as σ we can select the identity mapping. In this case the formula (3) has the form:

$$x \circ y = \begin{cases} x \cdot y & \text{for} \quad x, y \in Q, \; x \neq y, \\ x^2 & \text{for} \quad x \in Q, \; y = q, \\ y^2 & \text{for} \quad x = q, \; y \in Q, \\ q & \text{for} \quad x = y \in Q'. \end{cases} \tag{4}$$

If $(Q(\cdot)$ is an idempotent quasigroup, then (4) coincides with (2) and $Q'(\circ)$ is a loop with the identity q.

Example 2. The diagonal of the multiplication table of the additive group $\mathbb{Z}_3$ contains all elements of $\mathbb{Z}_3$. So, according to (4), the prolongation $\mathbb{Z}'_3(\circ)$ has the following multiplication table:

$\circ$	0	1	2	3
0	3	1	2	0
1	1	3	0	2
2	2	0	3	1
3	0	2	1	3

Putting $\alpha = \begin{pmatrix} 0\,1\,2\,3 \\ 3\,1\,2\,0 \end{pmatrix}$ and $x \odot y = \alpha(x \circ y)$ we can see that $\mathbb{Z}'_3(\circ)$ is isotopic to the Klein's group $K_4(\odot)$.

Using $\sigma = \begin{pmatrix} 0\,1\,2 \\ 2\,0\,1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 0\,1\,2 \\ 1\,2\,0 \end{pmatrix}$ we obtain two non-commutative prolongations:

$\circ$	0	1	2	3
0	0	1	3	2
1	3	2	0	1
2	2	3	1	0
3	1	0	2	3

$\circ$	0	1	2	3
0	0	3	2	1
1	1	2	3	0
2	3	0	1	2
3	2	1	0	3

These prolongations also are isotopic to the Klein's group. For the first we have $x \odot y = \alpha(x) \circ \beta(y)$, for the second $x \odot y = \beta(x) \circ \alpha(y)$, where $\alpha = \begin{pmatrix} 0\,1\,2\,3 \\ 0\,3\,2\,1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0\,1\,2\,3 \\ 0\,1\,3\,2 \end{pmatrix}$. $\square$

2. The construction proposed by G. B. Belyavskaya. This construction is valid for admissible quasigroups. At first we consider the case when $Q(\cdot)$ is an idempotent quasigroup. To find the prolongation $Q'(\diamond)$ of $Q(\cdot)$ we select an arbitrary element $a \in Q$. Next, in the multiplication table of $Q(\cdot)$ we replace all elements of the diagonal, except a, by q and adjunct one column and one row:

$$
\begin{array}{c|cccccc}
\cdot & 1 & 2 & \ldots & a & \ldots & n \\ \hline
1 & 1 & & & & & \\
2 & & 2 & & & & \\
\vdots & & & \ddots & & & \\
a & & & & a & & \\
\vdots & & & & & \ddots & \\
n & & & & & & n
\end{array}
\quad \longrightarrow \quad
\begin{array}{c|cccccc|c}
\diamond & 1 & 2 & \ldots & a & \ldots & n & q \\ \hline
1 & q & & & & & & 1 \\
2 & & q & & & & & 2 \\
\vdots & & & \ddots & & & & \vdots \\
a & & & & a & & & q \\
\vdots & & & & & \ddots & & \vdots \\
n & & & & & & q & n \\ \hline
q & 1 & 2 & \ldots & q & \ldots & n & a
\end{array}
$$

The operation in $Q(\diamond)$ is defined in the following way:

$$
x \diamond y = \begin{cases}
x \cdot y & \text{for} \quad x, y \in Q, \ x \neq y, \\
q & \text{for} \quad x = y \in Q - \{a\}, \\
a & \text{for} \quad x = y = a, \\
x & \text{for} \quad x \in Q - \{a\}, \ y = q, \\
y & \text{for} \quad x = q, \ y \in Q - \{a\}, \\
q & \text{for} \quad x = q, \ y = a, \\
q & \text{for} \quad x = a, \ y = q, \\
a & \text{for} \quad x = y = q.
\end{cases} \tag{5}
$$

In a general case, when $Q(\cdot)$ is "only" an admissible quasigroup, we can select a complete mapping σ of Q and fix an arbitrary element $a \in Q$. Then, obviously, there exists an uniquely determined element $x_a \in Q$ such that $a = x_a \cdot \sigma(x_a)$. The prolongation $Q'(\diamond)$ of $Q(\cdot)$ can be defined by

$$
x \diamond y = \begin{cases}
x \cdot y & \text{for} \quad x, y \in Q, \ y \neq \sigma(x), \\
q & \text{for} \quad x \in Q - \{x_a\}, \ y = \sigma(x), \\
a & \text{for} \quad x = x_a, \ y = \sigma(x_a), \\
\overline{\sigma}(x) & \text{for} \quad x \in Q - \{x_a\}, \ y = q, \\
\overline{\sigma}\sigma^{-1}(y) & \text{for} \quad x = q, \ y \neq \sigma(x_a), \\
q & \text{for} \quad x = q, \ y = \sigma(x_a), \\
q & \text{for} \quad x = x_a, \ y = q, \\
a & \text{for} \quad x = y = q.
\end{cases} \tag{6}
$$

Selecting different σ and different a we obtain different prolongations.

From a formal point of view, the above construction is a generalization on the classical construction. Indeed, putting $\sigma(q) = q$ we extend σ to a complete mapping of Q'. Next, putting $a = x_a = q$ in (6) we obtain (3).

If the diagonal of the multiplication table of $Q(\cdot)$ contains all elements of Q, then as σ can be selected the identity mapping and the formula (6) can be written in the form:

$$x \diamond y = \begin{cases} x \cdot y & \text{for} \quad x, y \in Q,\ x \neq y, \\ q & \text{for} \quad x = y \in Q - \{x_a\}, \\ a & \text{for} \quad x = y = x_a, \\ x^2 & \text{for} \quad x \in Q - \{x_a\},\ y = q, \\ y^2 & \text{for} \quad x = q,\ y \in Q - \{x_a\}, \\ q & \text{for} \quad x = q,\ y = x_a, \\ q & \text{for} \quad x = x_a,\ y = q, \\ a & \text{for} \quad x = y = q. \end{cases} \tag{7}$$

For idempotent quasigroups it coincides with (5) but, generally, prolongations obtained by the method proposed by G. B. Belyavskaya are not isotopic to prolongations obtained by the method proposed by V. D. Belousov. Below we present the corresponding example.

Example 3. The prolongation $\mathbb{Z}_3'(\diamond)$ of the additive group $\mathbb{Z}_3$ constructed according to (7), where $a = 1$, $x_a = 2$, $q = 3$, has the following multiplication table:

$\diamond$	0	1	2	3
0	3	1	2	0
1	1	3	0	2
2	2	0	1	3
3	0	2	3	1

This prolongation is isotopic to the group $\mathbb{Z}_4(+)$. The connection between $\mathbb{Z}_4(+)$ and $\mathbb{Z}_3'(\diamond)$ is given by the formula $\gamma(x+y) = \alpha(x) \diamond \alpha(y)$, where $\alpha = \begin{pmatrix} 0\,1\,2\,3 \\ 3\,1\,2\,0 \end{pmatrix}$, $\gamma = \begin{pmatrix} 0\,1\,2\,3 \\ 1\,2\,3\,0 \end{pmatrix}$. So, the prolongation of $\mathbb{Z}_3$ constructed by (7) and the prolongation of $\mathbb{Z}_3$ constructed by (4) (in Example 2) are not isotopic. $\square$

Example 4. Let $Q(\cdot)$ and σ be as in Example 1. Then, for example, for $a = 2$ we have $x_a = 3$. Whence, according to (6), we obtain the prolongation:

$\diamond$	1	2	3	4	5	6
1	1	2	3	6	5	4
2	4	6	1	5	2	3
3	2	5	4	1	3	6
4	5	4	2	3	6	1
5	3	1	6	2	4	5
6	6	3	5	4	1	2

Similarly, for $a = 3$ we have $x_a = 2$ and consequently

$\diamond$	1	2	3	4	5	6
1	1	2	3	6	5	4
2	4	3	1	5	2	6
3	6	5	4	1	3	2
4	5	4	2	3	6	1
5	3	1	6	2	4	5
6	2	6	5	4	1	3

Applying Theorem 2.5 from [10] we can verify that these prolongations are not isotopic to the prolongation obtained in Example 1. $\square$

4. Our construction

In the previous section methods of construction of a prolongation of quasigroups that have a complete mapping were given. But, as it is proved in [12] (see also [8], p. 36) there are quasigroups which do not possess such mappings. For example, a group of order $4k+2$ has no complete mapping.

Below, we give a new method of a construction of prolongations for quasigroups that have a quasicomplete mapping. Our method can also be applied to quasigroups that have a complete mapping.

Let $Q(\cdot)$ be an arbitrary quasigroup with a quasicomplete mapping σ. Then $|\overline{\sigma}(Q)| = n-1$ and $\mathrm{def}(\sigma) = d$ for some $d \in Q$. In this case we also have $\overline{\sigma}(x_1) = \overline{\sigma}(x_2) = a$, i.e., $x_1 \cdot \sigma(x_1) = x_2 \cdot \sigma(x_2) = a$ in $Q(\cdot)$, for some $x_1, x_2, a \in Q$, $x_1 \neq x_2$.

The idea of our construction is presented by the following tables, where for simplicity it is assumed that σ is the identity mapping and all elements of Q, except x_1 and x_2, are idempotents.

$$
\begin{array}{c|cccccccc}
\cdot & 1 & 2 & \dots & x_1 & \dots & x_2 & \dots & n \\
\hline
1 & 1 & & & & & & & \\
2 & & 2 & & & & & & \\
\vdots & & & \ddots & & & & & \\
x_1 & & & & a & & & & \\
\vdots & & & & & \ddots & & & \\
x_2 & & & & & & a & & \\
\vdots & & & & & & & \ddots & \\
n & & & & & & & & n \\
\hline
\end{array}
\quad \longrightarrow \quad
\begin{array}{c|cccccccc|c}
* & 1 & 2 & \dots & x_1 & \dots & x_2 & \dots & n & q \\
\hline
1 & q & & & & & & & & 1 \\
2 & & q & & & & & & & 2 \\
\vdots & & & \ddots & & & & & & \vdots \\
x_1 & & & & a & & & & & q \\
\vdots & & & & & \ddots & & & & \vdots \\
x_2 & & & & & & q & & & a \\
\vdots & & & & & & & \ddots & & \vdots \\
n & & & & & & & & q & n \\
\hline
q & 1 & 2 & \dots & q & \dots & a & \dots & n & d
\end{array}
$$

This new table is obtained from the old one by replacing all elements of the diagonal, except $a = x_1 \cdot x_1$, by q and adding one new row and column such that $x * q = q * x = x$ for $x \in Q - \{x_1, x_2\}$, $x_1 * q = q * x_1 = q$, $x_2 * q = q * x_2 = a$, $q * q = d$.

The operation of this new quasigroup is determined by the formula:

$$
x * y = \begin{cases}
x \cdot y & \text{for} \quad x, y \in Q,\ x \neq y, \\
q & \text{for} \quad x = y \in Q - \{x_1\}, \\
a & \text{for} \quad x = y = x_1, \\
x & \text{for} \quad x \in Q - \{x_1, x_2\},\ y = q, \\
y & \text{for} \quad x = q,\ y \in Q - \{x_1, x_2\}, \\
q & \text{for} \quad x = x_1,\ y = q \ \textit{ or } \ x = q,\ y = x_1, \\
a & \text{for} \quad x = x_2,\ y = q \ \textit{ or } \ x = q,\ y = x_2, \\
d & \text{for} \quad x = y = q.
\end{cases} \tag{8}
$$

In the general case, when σ is an arbitrary quasicomplete mapping of Q, $\operatorname{def}(\sigma) = d$, $a = \overline{\sigma}(x_1) = \overline{\sigma}(x_2)$, $x_1 \neq x_2$ and x_1 is fixed, the operation of $Q'(*)$ has the form:

$$
x * y = \begin{cases}
x \cdot y & \text{for} \quad x, y \in Q,\ y \neq \sigma(x), \\
q & \text{for} \quad x \in Q - \{x_1\},\ y = \sigma(x), \\
a & \text{for} \quad x = x_1,\ y = \sigma(x), \\
\overline{\sigma}(x) & \text{for} \quad x \in Q - \{x_1, x_2\},\ y = q, \\
\overline{\sigma}\sigma^{-1}(y) & \text{for} \quad x = q,\ y \neq \sigma(x_1),\ y \neq \sigma(x_2), \\
q & \text{for} \quad x = x_1,\ y = q \ \textit{ or } \ x = q,\ y = \sigma(x_1), \\
a & \text{for} \quad x = x_2,\ y = q \ \textit{ or } \ x = q,\ y = \sigma(x_2), \\
d & \text{for} \quad x = y = q.
\end{cases} \tag{9}
$$

If in the above formula we delete x_2 and assume that σ is a complete mapping, then for $x_1 = x_a$ and $d = a$ this formula will be identical with (7). This means that our construction is a generalization of the construction proposed by G. B. Belyavskaya. Consequently, it is also a generalization of the classical construction.

Example 5. Let $Q(\cdot)$ be a quasigroup defined in Example 1. The mapping $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 4\,5\,2\,3\,1 \end{pmatrix}$ is quasicomplete on Q, $\overline{\sigma} = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 4\,2\,5\,2\,3 \end{pmatrix}$ is its conjugated mapping, $\operatorname{def}(\sigma) = 1$, $\overline{\sigma}(2) = \overline{\sigma}(4) = 2$. Hence $d = 1$, $a = 2$, $x_1 = 2$, $x_2 = 4$. Putting $q = 6$ and using our construction we obtain the following prolongation of $Q(\cdot)$:

$*$	1	2	3	4	5	6
1	1	2	3	6	5	4
2	4	3	1	5	2	6
3	2	6	4	1	3	5
4	5	4	6	3	1	2
5	6	1	5	2	4	3
6	3	5	2	4	6	1

For $x_1 = 4$, $x_2 = 2$ our construction gives the quasigroup:

$*$	1	2	3	4	5	6
1	1	2	3	6	5	4
2	4	3	1	5	6	2
3	2	6	4	1	3	5
4	5	4	2	3	1	6
5	6	1	5	2	4	3
6	3	5	6	4	2	1

From Theorem 2.5 in [10] it follows that these two prolongations are isotopic, but they are not isotopic to the prolongation constructed in Example 1 and in Example 4. □

6. Conclusion

The Brualdi conjecture (cf. [8], p.103) says that each Latin square $n \times n$ possesses a sequence of $k \geqslant n-1$ distinct elements selected from different rows and different columns. In other words, each finite quasigroup has at least one complete or quasicomplete mapping. It is known that if a quasigroup $Q(\cdot)$ has a complete mapping, then each quasigroup isotopic to $Q(\cdot)$ has one also. Any group of odd order has a complete mapping, but, for example, groups of order $4k+2$ do not contain such mappings. More interesting facts on the Brualdi conjecture one can find in [1, 2, 8, 11] and [13].

If this conjecture is true, then from our results it follows that *each finite quasigroup has a prolongation.*

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