

## Galina B. Belyavskaya's 70th birthday



On April 19, 2010 Galina Borisovna Belyavskaya turned 70. Presently she is a leading researcher in the Institute of Mathematics and Computer Science of the Academy of Sciences of the Republic of Moldova. She has made a significant contribution to the development of binary and  $n$ -ary quasigroup theory and published about 70 research works in mathematical journals.

For more than 20 years Galina was the Scientific secretary of the Specialized Council for conferring scientific degree at the Institute of Mathematics of the Academy of Sciences of Moldova.

G. Belyavskaya was born in Ust'-Kamenogorsk, the capital of the East-Kazakhstan Region of former USSR (now Oskemen in Kazakhstan). Her parents, born in Altai Region (Siberia) were engineers. Her father worked as a trees rafter on Siberian rivers. Both were Russian. The father's mother

had re-married after her first husband died. G. Belyavskaya's father took polish sounding surname after her stepfather. Galina stayed with this surname. In the middle fifties the family moved to Gomel (Belarus) and then to Kishinev where her father became a teacher.

After her father she also inherited the love to play chess. In her youth she played chess and became a junior chess champion of Moldova.

In 1957 she began study at the Faculty of Physics and Mathematics of the Kishinev State University which she graduated with honors in 1962. In the same year she joined the newly established Institute of Mathematics of the Academy of Sciences of the Republic of Moldova, that was then a branch of the Academy of Sciences of the USSR. She still works there today.

Initially she worked in the computer laboratory and develop new programming languages, algorithms and software. Her paper [1] is from that period. In this paper one simple criterium for classifications of partially symmetric boolean functions is presented.

Since 1967, Belyavskaya begins cooperation with V. D. Belousov. Her first papers devoted to quasigroups are connected with the problem of a prolongation (extension) of quasigroups, i.e., a construction of a quasigroup on  $(n + 1)$ -th order from a quasigroup of  $n$ -th order, and with the problem of a contraction (compression), i.e., a construction of a quasigroups of  $n$ -th order from quasigroups of  $(n + 1)$ -th order. Necessary and sufficient conditions under which two contraction of a given quasigroup are isotopic are found in [2] and [3]. A new method of a prolongation is presented in [5]. Necessary and sufficient condition of isotopy of such two prolongations of a given quasigroup are found too. The problem of construction and decomposition of quasigroups was investigated in many of her papers (cf. [12], [27] and [33]).

Next she studied the systems of binary operations containing two projections, all quasigroup operations defined on a fixed set  $Q$  and satisfying the generalized Stein's identity ([7], [8] and [13]). Properties of such systems are described by means of balanced incomplete block design. A method for constructing such systems is presented in [7]. Later she generalized those results to the systems of  $n$ -ary quasigroup operations (see [49] and [61]).

Many papers of G. B. Belyavskaya are connected with the problem of orthogonality of binary and  $n$ -ary quasigroups. She start with a characterization of  $r$ -orthogonal quasigroups, i.e., quasigroups  $Q(\cdot)$ ,  $Q(\circ)$  for which the set  $\{(x \cdot y, x \circ y) : x, y \in Q\}$  contains exactly  $r$  different ordered pairs. In [16] it is proved that for any  $n \geq 4$  there exist  $(n+k)$ -orthogonal quasigroups

for any  $k$  with  $2 \leq k \leq [n/2]$ . Necessary and sufficient conditions for a finite quasigroup to have an  $r$ -orthogonal quasigroup are found in [17]. Abelian groups of order  $n > 2$ ,  $n \neq 4$ , have no  $(n^2 - 2)$ -,  $(n^3 - 3)$ - or  $(n^2 - 5)$ -orthogonal quasigroups. Groups of prime order  $n$  have no  $(n+2)$ -,  $(n+3)$ -,  $(n+4)$ - or  $(n+5)$ -orthogonal quasigroups. A method of construction of  $(n^2 - 2)$ -orthogonal quasigroups of even order  $n$ , where  $n \not\equiv 1 \pmod{3}$ , by means of extensions of abelian groups is given in [21]. The set of possible values of  $r$  for which there exist pairs of  $r$ -orthogonal quasigroups of order  $n$  is described in [23], [25] and [37]. The class of self-orthogonal  $n$ -ary groupoids is characterized in [31]; pairwise orthogonality of  $n$ -ary operations in [53].

A new and more general version of orthogonality for  $n$ -ary operations is presented in [53] and [57]. It is connected with hypercubes which are a generalization of Latin squares to higher dimensions.

A series of her papers is devoted to admissible quasigroups  $Q(\cdot)$ , i.e., quasigroups with  $m$  elements containing a sequence of  $m$  elements from different rows and columns of the multiplication table of  $Q(\cdot)$ . If this sequence has exactly  $t$  distinct elements, then we say that a quasigroup  $Q(\cdot)$  is  $t$ -admissible. The main results of Belyavskaya on such quasigroups are contained in [15], [18] and [24]. For example, all numbers  $t$  such that a cyclic group  $G$  is  $t$ -admissible are determined in [15]. For an arbitrary finite group similar result is obtained in [24]. Admissible  $n$ -ary quasigroups are studied in [19], [20] and [22].

In the early seventies of last century Belyavskaya investigated semisymmetric Stein quasigroups, for which she proved that a semisymmetric Stein quasigroup is invariant under parastrophy [9]. In this paper she also shows that a semisymmetric Stein quasigroup is isotopic to a group if and only if it is distributive.

In the late eighties Belyavskaya's scientific interest has been focused on the study of algebraic problems of quasigroups. In that time she introduced several new concepts and has received many important results. To the most important concepts should be included the concept of *chain isotopic quasigroups* [4], the concept of the *centre* and the new concept of *nuclei* that have led to many significant results (cf. [29], [30], [34], [36], [40], [41]). *Commutators* and *associators* of quasigroups introduced and described by her (cf. [44], [45], [46] and [47]) are useful during investigations of quasigroups.

A large cycle of her works is devoted to  $T$ -quasigroups and quasigroups which are *linear* or *alinear over groups* (cf. [38], [39], [42] and [43]). The

characterization of  $T$ -quasigroups, linear and alinear quasigroups with the help of identities is one of the most important results in the theory of quasigroups which are linear over groups.

The last papers of G. Belyavskaya are connected with universal-algebraic problems of the theory of quasigroups and with application of binary and  $n$ -ary quasigroups in coding theory. In [65] she suggest a general method of the construction of secret-sharing schemes based on orthogonal systems of partial (in particular, everywhere determined)  $k$ -ary operations which generalizes some known methods of the construction of such schemes by finite fields and point out the orthogonal systems of  $k$ -ary operations respective of these known schemes.

Galina Belyavskaya was a supervisor of five PhD thesis (S. Murathudjaev, A. Lumpov, P. Syrbu, L. Ursu, A. Tabarov). Many scientists from Moldova and other countries were trained under her supervision. She was the scientific adviser of graduate students from the Kishinev State University.

Since 1971 G. B. Belyavskaya was the assistant of V. D. Belousov in the sector of the theory of quasigroups. After his death she has headed the research team of the theory of quasigroups at the Institute of Mathematics of the Academy of Sciences of Moldova.

She is an Advisory Editor of the international journal *Quasigroups and Related Systems*, and also a member of the Editorial Board of the *Buletinul Academiei de Ştiinţe a Republicii Moldova, Matematica*.

G. B. Belyavskaya is kind, sympathetic, delicate, trustworthy, very disciplined, honest and modest woman. She is a good wife, mother, grandmother and great grandmother. Recently she has became interested in esoteric and she published two books on this topic.

Dear Galina Borisovna: The authors of this note heartily congratulate you on your 70th birthday and wish you continuing success in your scientific and pedagogical work, strong health, and many long years of life. Thank you for all that you have done for us.

*Wieslaw A. Dudek  
Victor Shcherbacov*

Below we present the full list of publications of Galina B. Belyavskaya. English translations of Russian titles as given in *Mathematical Reviews* and *Zentralblatt für Mathematik* may be somewhat different from those used in this list.

### List of publications of Galina B. Belyavskaya

1. *Accounting of partial symmetry of Boolean functions in the synthesis of logical schemes*, (with Yu. N. Pecherskij), (Russian), Teor. Diskret. Avtomatov, Akad. Nauk Latv. SSR (1967), 51 – 54.
2. *Contraction of quasigroups, I*, (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **1** (1970), 6 – 12.
3. *Contraction of quasigroups, II*, (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **3** (1970), 3 – 17.
4. *Chain-isotopic quasigroups*, (Russian), Mat. Issled. **5** (1970), 13 – 27.
5. *On generalized prolongation of quasigroups*, (Russian), Mat. Issled. **5**(1970), 28 – 48.
6. *Algorithms for the solution of certain problems in the theory of quasigroups*, (Russian), in "Voprosy teorii kvazigrupp i lup", Kishinev 1970, 20 – 30.
7. *S-systems of an arbitrary index, I*, (with A. M. Cheban), (Russian), Mat. Issled. **7** (1972), 27 – 43.
8. *S-systems of an arbitrary index, II*, (with A. M. Cheban), (Russian), Mat. Issled. **7** (1972), no. 2, 3 – 13.
9. *On semisymmetric Stein's quasigroups*, (with A. M. Cheban), (Russian), Mat. Issled. **7** (1972), no. 3, 231 – 237.
10. *Isotopy of A-quasigroups*, (with M. D. Kitoroage), (Russian), Mat. Issled. **8** (1973), no. 1, 3 – 19.
11. *On a representation of composition of n-ary operations by trees*, (with A. M. Cheban), (Russian), in "Issled. teorii kvazigrupp i lup", Kishynev 1973, 52 – 58.
12. *Wreath product of quasigroups by means of pairwise balanced block designs*, (Russian), in "Kombinatornyi analiz", Moscov MGU, 1974, 49 – 53.
13. *S-systems of quasigroups*, (Russian), Mat. Issled. **9** (1974), no. 2, 10 – 18.
14. *Interdependence of certain closure conditions in k-nets*, (with V. D. Belousov), (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **2** (1974), 44 – 51.

15. *On admissibility of quasigroups*, (with A. F. Russu), (Russian), Mat. Issled. **10** (1975), no. 1(35), 45 – 57.
16. *r-orthogonal quasigroups, I*, (Russian), Mat. Issled. **39** (1976), 32 – 39.
17. *r-orthogonal quasigroups, II*, (Russian), Mat. Issled. **43** (1977), 39 – 49.
18. *On partially admissible quasigroups*, (with A. F. Russu), (Russian), Mat. Issled. **43** (1977), 50 – 58.
19. *Admissible n-ary quasigroups, I*, (with S. Murathudjaev), (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **2** (1977), 14 – 21.
20. *About admissibility of n-ary quasigroups*, (with S. Murathudjaev), Proc. Colloq. Math. Soc. J. Bolyai 18, Keszthely, Hungary, 1976, vol.1 "Combinatorics", North-Holland 1978, 101 – 119.
21. *Construction of (n-2)-orthogonal quasigroups of even order n, where n-1 ≠ 0(mod 3)*, (Russian), Mat. Issled. **51** (1979), 23 – 26.
22. *Admissible n-ary quasigroups, II*, (with S. Murathudjaev), (Russian), Mat. Issled. **51** (1979), 27 – 39.
23. *On the spectrum of partial orthogonality of quasigroups of lower orders*, (Russian), Mat. Issled. **66** (1982), 7 – 14.
24. *Spectrum of partial admissibility of finite quasigroups (Latin squares)*, (Russian), Mat. Zamietki **32** (1982), 777 – 788 (translation in Math. Notes **32** (1983), 874 – 880).
25. *On partially orthogonal quasigroups and systems of quasigroups*, (Russian), Mat. Issled. **71** (1983), 25 – 33.
26. *Cross product of two systems of quasigroups and its use in constructing partially orthogonal quasigroups*, with A. D. Lumpov), (Russian), Mat. Issled. **83** (1985), 26 – 38.
27. *Direct decompositions of quasigroups*, (Russian), Mat. Issled. **95** (1987), 23 – 38.
28. *Completion of a group and the construction of orthogonal quasigroups of order  $3t + i$ ,  $i = 0, 1, 2$ ,  $t ≠ 2, 6$ , with orthogonal subquasigroups of order  $t$* , (with A. V. Nazarok), (Russian), Mat. Issled. **95** (1987), 39 – 52.
29. *The nuclei and center of a quasigroups*, (Russian), Mat. Issled. **102** (1988), 37 – 52.
30. *T-quasigroups and the center of a quasigroup*, (Russian), Mat. Issled. **111** (1989), 24 – 43.
31. *On one class of self-orthogonal n-groupoids*, (with P. N. Syrbu), (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **2** (1989), 25 – 30.

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32. *Latin squares, quasigroups and their applications*, (with V. D. Belousov), (Russian), Ştiinţa, Kishinev, 1989, ISBN: 5-376-00074-5.
  33. *Complete direct decompositions of quasigroups with an idempotent*, (Russian), Mat. Issled. **113** (1990), 21 – 36.
  34. *On the concept of a center in a quasigroup*, (Russian), Mat. Issled. **120** (1991), 8 – 17.
  35. *On one equivalence in quasigroups*, (with A. D. Lumpov), (Russian), Mat. Issled. **120** (1991), 18 – 29.
  36. *The nuclei and center of linear quasigroups*, (with A. Th. Tabarov), (Russian), Izv. Akad. Nauk Moldav. SRSR, ser. Fiz.-Tehn. Mat. Nauk **3** (1991), 37 – 42.
  37. *r-orthogonal Latin squares*, Annals Discr. Math. **46** (1991), 169 – 202.
  38. *Characterization of linear and alinear quasigroups*, (with A. Th. Tabarov), (Russian), Diskr. Mat. **4** (1992), 142 – 147.
  39. *Autotopies determining isotopies and autotomorphism of T-quasigroups*, (with A. Th. Tabarov), (Russian), in "Algebraic methods in geometry", Univ. Druzhby Narodov, Moskva, 1992, 77 – 82.
  40. *Centre and multiplication groups of quasigroups*, Bul. Acad. Ştiinţe Repub. Mold., Mat. **2(8)** (1992), 81 – 89.
  41. *On concept of centre in quasigroups*, Demonstratio Math. **26** (1993), 75–84.
  42. *Abelian quasigroups are T-quasigroups*, Quasigroups and Related Systems **1** (1994), 1 – 7.
  43. *One-sided T-quasigroups and irreducible balanced identities*, Quasigroups and Related Systems **1** (1994), 8 – 21.
  44. *Associators, commutators and linearity of a quasigroups*, Discrete Math. Appl. **5** (1995), 577 – 586.
  45. *Theory of quasigroups: nuclei, center and commutator*, (Russian), Bul. Acad. Ştiinţe Repub. Mold., Mat. **2(21)** (1996), 47 – 71.
  46. *Associants and the commutant of a quasigroup*, (Russian), Fundam. Prikl. Mat. **3** (1997), 715 – 737.
  47. *On commutators of quasigroup congruences*, Bul. Acad. Ştiinţe Rep. Mold., Mat. **2(27)** (1998), 91 – 101.
  48. *Centrally isotopic quasigroups*, Quasigroups and Related Systems **5** (1998), 1 – 12.
  49. *Quasigroup power sets and cyclic S-systems*, Quasigroups and Related Systems **9** (2002), 1 – 17.

50. *Check character systems over quasigroups and loops*, (with V. I. Izbash and V. A. Shcherbacov), *Quasigroups and Related Systems* **10** (2003), 1 – 28.
51. *On check character systems over quasigroups and loops*, *Algebra Discrete Math.* **2** (2003), 1 – 13.
52. *On check character systems over groups*, (with A. Diordiev), *Bul. Acad. Ştiinţe Repub. Mold., Mat.* **3(46)** (2004), 17 – 24.
53. *Pairwise orthogonality of  $n$ -ary operations*, *Bul. Acad. Ştiinţe Repub. Mold., Mat.* **3(49)** (2005), 5 – 18.
54. *On some quasi-identities in finite quasigroups*, (with A. Diordiev), *Bul. Acad. Ştiinţe Repub. Mold., Mat.* **3(49)** (2005), 19 – 32.
55. *Check character systems using quasigroups, I*, (with V. I. Izbash and G. L. Mullen), *Designs, Codes Cryptography* **37** (1995), 215 – 227.
56. *Check character systems using quasigroups, II*, (with V. I. Izbash and G. L. Mullen), *Designs, Codes Cryptography* **37** (1995), 405 – 419.
57. *Orthogonal hypercubes and  $n$ -ary operations*, *Quasigroups and Related Systems* **13** (2005), 73 – 86.
58. *Strongly orthogonal and uniformly orthogonal many-place operations*, (with G. L. Mullen), *Algebra Discrete Math.* **1** (2006), 1 – 17.
59. *Power sets of  $n$ -ary quasigroups*, *Bul. Acad. Ştiinţe Republ. Moldova, Mat.* **1(53)** (2007), 37 – 45.
60. *Identities with permutations associated with quasigroups isotopic to groups*, *Bul. Acad. Ştiinţe Republ. Moldova, Mat.* **2(53)** (2007), 19 – 24.
61.  *$S$ -systems of  $n$ -ary quasigroups*, *Quasigroups and Related Systems* **15** (2007), 251 – 260.
62. *On groupoids with the identity defining commutative Moufang loops*, (with A. Tabarov), (Russian), *Fund. Prik. Mat.* **14** (2008), 33 – 39.
63. *Conjugate-orthogonality and complete multiplication groups of quasigroups*, (with A. Diordiev), *Bul. Acad. Ştiinţe Republ. Moldova, Mat.* **1(59)** (2008), 22 – 30.
64. *Identities with permutations leading to the linearity of quasigroups*, *Discrete Math. Appl.* **19** (2009), 173 – 190.
65. *Secret-sharing schemes and orthogonal systems of  $k$ -ary operations*, *Quasigroups and Related Systems* **17** (2009), 161 – 176.
66. *Check character systems and totally conjugate orthogonal  $T$ -quasigroups*, *Quasigroups and Related Systems* **18** (2010), 7 – 16.
67. *Polynomial  $k$ -ary operations, matrices, and  $k$ -mappings*, *J. Gen. Lie Theory Appl.* **4** Article ID 100301.