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Errata to "Shortest single axioms with neutral element for groups of exponent 2 and 3"

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The purpose of this note is to correct an error in [1] pointed out in [2] and to explain the source of this error. Upon initial review of [1], one reviewer requested that multiplicative dots be used to reduce the large number of parentheses in the identities. For instance, $(ey)((((yy)(xz))z)z) = x$ would be changed to $ey \cdot ((yy \cdot xz)z \cdot z) = x$. A program was written to facilitate this conversion for the the large number of identities contained in Theorems 6.1 and 6.2 of [1]. Given that the right hand side of every identity in Theorem 6.1 should be x, the right hand side of every identity in Theorem 6.2 was inadvertently changed to x as well. For example, $e((x(xy)z)y) = z$ became $e(x(xy \cdot z) \cdot y) = x$ instead of the correct $e(x(xy \cdot z) \cdot y) = z$. Below is the corrected version of Theorem 6.2 of [1].

Theorem. *The following* 173 *identities with neutral element* (*and their mirrors*) *are single axioms with neutral element for Boolean groups.*

$e(x(xy \cdot z) \cdot y) = z$	$ex \cdot (xy \cdot zy) = z$	$e(x \cdot y(x \cdot zy)) = z$
$e(xy \cdot (x \cdot zy)) = z$	$ex \cdot y(x \cdot zy) = z$	$(e \cdot xy)(y \cdot zx) = z$
$e((x \cdot yz) \cdot xy) = z$	$ex \cdot y(z \cdot xy) = z$	$e(x \cdot y(z \cdot yx)) = z$
$e((x \cdot yz)y \cdot x) = z$	$ex \cdot y(z \cdot yx) = z$	$e(x \cdot yz) \cdot yx = z$
$(e \cdot (x \cdot yz)z)x = y$	$x(ey \cdot (x \cdot zy)) = z$	$xe \cdot y(x \cdot zy) = z$
$xe \cdot (yx \cdot z)y = z$	$(xe \cdot y)(x \cdot zy) = z$	$(x \cdot (ey \cdot x)z)y = z$
$(x \cdot (ey \cdot x)z)z = y$	$(xe \cdot (yx \cdot z))z = y$	$xe \cdot y(yz \cdot x) = z$
$x(ey \cdot (z \cdot xy)) = z$	$x((e \cdot yz) \cdot xy) = z$	$x(ey \cdot z) \cdot xy = z$
$x((e \cdot yz)x \cdot z) = y$	$xe \cdot (y \cdot zx)z = y$	$xe \cdot (yz \cdot x)z = y$
$((xe \cdot y) \cdot zx)z = y$	$((x \cdot ey)z \cdot x)z = y$	$((xe \cdot y)z \cdot x)z = y$
$x(e \cdot y(zy \cdot x)) = z$	$x((ey \cdot z) \cdot yx) = z$	$(xe \cdot y)(zy \cdot x) = z$
$x(ey \cdot z) \cdot yx = z$	$(x(e \cdot yz) \cdot y)x = z$	$((xe \cdot y)z \cdot y)x = z$
$(x(e \cdot yz) \cdot z)x = y$	$(xe \cdot yz)z \cdot x = y$	$x(x(ey \cdot z) \cdot y) = z$
$x \cdot (xe \cdot yz)y = z$	$x(x(y \cdot ez) \cdot y) = z$	$x(y \cdot e(x \cdot zy)) = z$
$x(ye \cdot (x \cdot zy)) = z$	$xy \cdot (ex \cdot z)y = z$	$x(y \cdot (e \cdot yz)x) = z$
$x(ye \cdot (yz \cdot x)) = z$	$xy \cdot (ey \cdot z)x = z$	$(x \cdot ye)(y \cdot zx) = z$
$(xy \cdot (e \cdot yz))x = z$	$((x \cdot ye) \cdot yz)x = z$	$x(y \cdot e(z \cdot xy)) = z$
$(x \cdot ye)(zx \cdot y) = z$	$x(y \cdot ez) \cdot xy = z$	$((x \cdot ye)z \cdot x)y = z$
$(xy \cdot ez) \cdot xz = y$	$(x \cdot ye)z \cdot xz = y$	$(xy \cdot ez)x \cdot z = y$
$x(y \cdot ez) \cdot yx = z$	$((x \cdot ye) \cdot zy)x = z$	$x((y \cdot ez) \cdot zx) = y$
$(xy \cdot ez) \cdot zx = y$	$(x(y \cdot ez) \cdot z)x = y$	$e(x \cdot (x \cdot yz)y) = z$
$(ex \cdot (xy \cdot z))y = z$	$e(x \cdot (yx \cdot z)y) = z$	$e(xy \cdot (xz \cdot y)) = z$
$ex \cdot (yx \cdot zy) = z$	$(e \cdot xy)(x \cdot zy) = z$	$(e \cdot x(y \cdot xz))y = z$
$e(xy \cdot xz) \cdot y = z$	$(ex \cdot (yx \cdot z))y = z$	$((e \cdot xy) \cdot xz)y = z$
$(ex \cdot (y \cdot xz))z = y$	$(ex \cdot (yx \cdot z))z = y$	$e(xy \cdot (yz \cdot x)) = z$

$$\begin{array}{rrr}
ex\cdot y(yz\cdot x)=z & (ex\cdot y)(yz\cdot x)=z & ((e\cdot xy)\cdot yz)z=x \\
e(x\cdot(yz\cdot x)y)=z & ex\cdot(yz\cdot xy)=z & (e\cdot x(y\cdot zx))y=z \\
e((xy\cdot z)x\cdot z)=y & ex\cdot(yz\cdot xz)=y & ex\cdot(yz\cdot x)z=y \\
(e(xy\cdot z)\cdot x)z=y & e(x\cdot y(zy\cdot x))=z & e((x\cdot yz)\cdot yx)=z \\
ex\cdot(yz\cdot yx)=z & (ex\cdot yz)\cdot yx=z & e((x\cdot yz)y\cdot z)=x \\
e(xy\cdot z)\cdot yz=x & e(xy\cdot zy)\cdot z=x & ((e\cdot xy)\cdot zy)z=x \\
(e(x\cdot yz)\cdot y)z=x & (ex\cdot yz)z\cdot x=y & e((x\cdot yz)z\cdot y)=x \\
e(xy\cdot z)\cdot zy=x & x\cdot e(yx\cdot zy)=z & x((ey\cdot x)\cdot zy)=z \\
x((e\cdot yx)z\cdot y)=z & (x\cdot ey)(xz\cdot y)=z & (x\cdot e(yx\cdot z))y=z \\
x(ey\cdot xz)\cdot y=z & (xe\cdot(y\cdot xz))y=z & ((xe\cdot y)\cdot xz)y=z \\
xe\cdot(y\cdot xz)z=y & (x\cdot e(yx\cdot z))z=y & ((xe\cdot y)\cdot xz)z=y \\
x(e\cdot y(yz\cdot x))=z & x\cdot e(yz\cdot xy)=z & x(e\cdot(yz\cdot x)y)=z \\
x((ey\cdot z)\cdot xy)=z & x((e\cdot yz)x\cdot y)=z & (x\cdot e(y\cdot zx))y=z \\
((xe\cdot y)\cdot zx)y=z & x(e\cdot(yz\cdot x)z)=y & (xe\cdot yz)\cdot xz=y \\
x\cdot e(yz\cdot yx)=z & x(ey\cdot(z\cdot yx))=z & (xe\cdot yz)\cdot zx=y \\
x((x\cdot ey)\cdot zy)=z & x((xe\cdot y)z\cdot y)=z & (x\cdot(x\cdot ey)z)y=z \\
x(xy\cdot(e\cdot zy))=z & x(x(ye\cdot z)\cdot y)=z & x\cdot(xy\cdot ez)y=z \\
x((xy\cdot e)z\cdot y)=z & x(xy\cdot ez)\cdot y=z & xy\cdot e(x\cdot zy)=z \\
xy\cdot e(xz\cdot y)=z & (xy\cdot e)(x\cdot zy)=z & (x\cdot y(e\cdot xz))y=z \\
x(ye\cdot xz)\cdot y=z & (xy\cdot(e\cdot xz))y=z & ((xy\cdot e)\cdot xz)y=z \\
x(y\cdot(ey\cdot z)x)=z & xy\cdot e(y\cdot zx)=z & xy\cdot e(yz\cdot x)=z \\
(xy\cdot e)(y\cdot zx)=z & ((xy\cdot e)\cdot yz)x=z & x((y\cdot ez)x\cdot y)=z \\
(x\cdot y(e\cdot zx))y=z & x((y\cdot ez)\cdot xz)=y & x\cdot(ye\cdot zx)z=y \\
x((y\cdot ez)x\cdot z)=y & (x\cdot y(e\cdot zx))z=y & (x\cdot y(ez\cdot x))z=y \\
x(ye\cdot zx)\cdot z=y & ((xy\cdot e)\cdot zx)z=y & x(y\cdot e(zy\cdot x))=z \\
x(y\cdot(ez\cdot y)x)=z & (x(y\cdot ez)\cdot y)x=z & x(ye\cdot z)\cdot zx=y \\
e(x\cdot(xy\cdot z)y)=z & e(x(yx\cdot z)\cdot y)=z & (ex\cdot(y\cdot xz))y=z \\
e(x(yx\cdot z)\cdot z)=y & ((e\cdot xy)\cdot yz)x=z & (ex\cdot yz)\cdot xy=z \\
(e(x\cdot yz)\cdot y)x=z & x(ye\cdot z)\cdot xz=y & e((xy\cdot z)\cdot yz)=x \\
x(e(x\cdot yz)\cdot y)=z & x(e(xy\cdot z)\cdot y)=z & xe\cdot(x\cdot yz)y=z \\
x\cdot e(xy\cdot zy)=z & x\cdot(ex\cdot yz)y=z & x((e\cdot xy)z\cdot y)=z \\
xe\cdot(xy\cdot z)y=z & (x\cdot e(xy\cdot z))y=z &
\end{array}$$

A finite model of any of the following 5 *identities with neutral element* (*or their mirrors*) *is a Boolean group with neutral element* e.

$$\begin{array}{rrr}
(ex\cdot y)z\cdot xz=y & ((ex\cdot y)z\cdot x)z=y & (ex\cdot y)z\cdot yz=x \\
(ex\cdot yz)y\cdot z=x & (ex\cdot yz)z\cdot y=x &
\end{array}$$

Any additional single axioms with neutral element for Boolean groups are among these 5 *identities with neutral element* (*up to renaming, mirroring, and symmetry*).

References

[1] **N. C. Fiala and K. M. Agre**, *Shortest single axioms with neutral element for groups of exponent* 2 *and* 3, Quasigroups and Related Systems **21** (2013), 69 – 82.

[2] **A. Krapež**, *Fiala–Agre list of single axioms for Boolean groups is wrong*, Quasigroups and Related Systems **22** (2014), 255 – 256.

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