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On quasi n-absorbing elements of multiplicative lattices

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Abstract. A proper element q of a lattice L is said to be a quasi n-absorbing element if whenever $a^n b \leqslant q$ implies that either $a^n \leqslant q$ or $a^{n-1}b \leqslant q$. We investigate properties of this new type of elements and obtain some relations among prime, 2-absorbing, n-absorbing elements in multiplicative lattices.

1. Introduction

In this paper we define and study quasi n-absorbing elements in multiplicative lattices. A *multiplicative lattice* is a complete lattice L with the least element 0 and compact greatest element 1, on which there is defined a commutative, associative, completely join distributive product for which 1 is a multiplicative identity. Notice that $L(R)$ the set of all ideals of a commutative ring R is a special example for multiplicative lattices which is principally generated, compactly generated and modular. However, there are several examples of non-modular multiplicative lattices (see [1]). Weakly prime ideals [3] were generalized to multiplicative lattices by introducing weakly prime elements [7]. While 2-absorbing, weakly 2-absorbing and n-absorbing ideals in commutative rings were introduced in [5], [6], and [4], 2-absorbing and weakly 2-absorbing elements in multiplicative lattices were studied in [10].

We begin by recalling some background material which will be needed. An element a of L is said to be *compact* if whenever $a \leqslant \bigvee_{\alpha\in I} a_\alpha$ implies $a \leqslant \bigvee_{\alpha\in I_0} a_\alpha$ for some finite subset I_0 of I. By a *C-lattice* we mean a (not necessarily modular) multiplicative lattice which is generated under joins by a multiplicatively closed subset C of compact elements of L. We note that in a C-lattice, a finite product of compact elements is again compact. Throughout this paper L and L_* denotes a multiplicative lattice and the set of compact elements of the lattice L, respectively. An element a of L is said to be *proper* if $a < 1$. A proper element p of L is said to be *prime* (resp. *weakly prime*) if $ab \leqslant p$ (resp. $0 \neq ab \leqslant p$) implies either $a \leqslant p$ or $b \leqslant p$. If 0 is prime, then L is said to be a *domain*. A proper element m of L is said to be *maximal* if $m < x \leqslant 1$ implies $x = 1$. The *Jacobson radical* of a lattice L is

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defined as $J(L) = \bigwedge\{m \mid m$ is a maximal element of $L\}$. L is said to be *quasi-local* if it contains a unique maximal element. If $L = \{0, 1\}$, then L is called a field. For $a \in L$, we define a *radical* of a as $\sqrt{a} = \bigwedge\{p \in L \mid p$ is prime and $a \leqslant p\}$. Note that in a C-lattice L,

$$\sqrt{a} = \bigwedge\{p \in L \mid p \text{ is prime and } a \leqslant p\} = \bigvee\{x \in L_* \mid x^n \leqslant a \text{ for some } n \in Z^+\}$$

by (Theorem 3.6 of [12]). Elements of the set $Nil(L) = \sqrt{0}$ are called *nilpotent*. For any prime element $p \in L$ by L_p we denote the localization $F = \{x \in C \mid x \nleqslant p\}$. For details on C-lattices and their localizations see [9] and [11]. An element $e \in L$ is said to be *principal* [8], if it satisfies the identities (i) $a \wedge be = ((a : e) \wedge b)e$ and (ii) $(ae \vee b) : e = (b : e) \vee a$. Elements satisfying the identity (i) are called *meet principal*, elements satisfying (ii) are called *join principal*. Note that any finite product of meet (join) principal elements of L is again meet (join) principal [8, Lemma 3.3 and Lemma 3.4]. If every element of L is principal, then L is called a *principal element lattice* [2].

Recall from [10] that a proper element q of L is called 2-*absorbing* (resp. *weakly* 2-*absorbing*) if whenever $a, b, c \in L$ with $abc \leqslant q$ (resp. $0 \neq abc \leqslant q$), then either $ab \leqslant q$ or $ac \leqslant q$ or $bc \leqslant q$. We say that (a, b, c) is a *triple zero element* of q if $abc = 0$, $ab \nleqslant q$, $ac \nleqslant q$ and $bc \nleqslant q$. Observe that if q is a weakly 2-absorbing element which is not a 2-absorbing, then there exist a triple zero of q. A proper element $q \in L$ is n-*absorbing* (resp. *weakly* n-*absorbing*) if $a_1a_2 \cdots a_{n+1} \leqslant q$ (resp. $0 \neq a_1a_2 \cdots a_{n+1} \leqslant q$) for some $a_1a_2 \cdots a_{n+1} \in L_*$ then $a_1a_2 \cdots a_{k-1}a_{k+1} \cdots a_{n+1} \leqslant q$ for some $k = 1, \ldots, n+1$.

2. Quasi n-absorbing elements

Let L be a multiplicative lattice and n be a positive integer.

Definition 2.1. A proper element q of L is called:
- *quasi* n-*absorbing* if $a^n b \leqslant q$ for some $a, b \in L_*$ implies $a^n \leqslant q$ or $a^{n-1}b \leqslant q$,
- *weakly quasi* n-*absorbing* if $0 \neq a^n b \leqslant q$ for some $a, b \in L_*$ implies $a^n \leqslant q$ or $a^{n-1}b \leqslant q$.

Theorem 2.2. *Let q be a proper element of L and $n \geqslant 1$. Then:*
(1) *q is a prime element if and only if it is quasi* 1-*absorbing,*
(2) *q is a weakly prime element if and only if it is weakly quasi* 1-*absorbing,*
(3) *if q is n-absorbing, then it is quasi n-absorbing,*
(4) *if q is quasi n-absorbing, then it is weakly quasi n-absorbing,*
(5) *if q is quasi n-absorbing, then it is quasi m-absorbing for all $m \geqslant n$,*
(6) *if q is weakly quasi n-absorbing, then it is weakly quasi m-absorbing for all $m \geqslant n$.*

Proof. (1), (2), (3) and (4) are obvious. To prove (5) suppose that q is a quasi n-absorbing element of L, and $a, b \in L_*$ with $a^m b \leqslant q$ for some $m \geqslant n$. Hence

$a^n(a^{m-n}b) \leqslant q$. Since q is a quasi n-absorbing element, we have either $a^n \leqslant q$ or $a^{n-1}(a^{m-n}b) \leqslant q$. So, either $a^m \leqslant q$ or $a^{m-1}b \leqslant q$. This shows that q is a quasi m-absorbing element of L.

(6) can be proved analogously. □

Corollary 2.3. *Let q be a proper element of L.*

(1) *If q is prime, then it is quasi n-absorbing for all $n \geqslant 1$.*

(2) *If q is weakly prime, then it is weakly quasi n-absorbing all $n \geqslant 1$.*

(3) *If q is 2-absorbing, then it is a quasi n-absorbing for all $n \geqslant 2$.*

(4) *If q is weakly 2-absorbing, then it is weakly quasi n-absorbing for all $n \geqslant 2$.*

The converses of these relations are not true in general.

Example 2.4. Consider the lattice of ideals of the ring of integers $L = L(\mathbb{Z})$. Note that the element $30\mathbb{Z}$ of L is a quasi 2-absorbing element, and so quasi n-absorbing element for all $n \geqslant 2$ by Corollary 2.3, but it is not a 2-absorbing element of L by Theorem 2.6 in [7].

Proposition 2.5. *For a proper element q of L the following statements are equivalent.*

(1) *q is a quasi n-absorbing element of L.*

(2) *$(q : a^n) = (q : a^{n-1})$ where $a \in L_*$, $a^n \nleqslant q$.*

In paticular, 0 is a quasi n-absorbing element of L if and only if for each $a \in L_*$ we have $a^n = 0$ or $ann(a^n) = ann(a^{n-1})$.

Proof. It follows directly from Definition 2.1. □

Notice that if q is a weakly quasi n-absorbing element which is not quasi n-absorbing, then there are some elements $a, b \in L_*$ such that $a^n b = 0$, $a^n \nleqslant q$ and $a^{n-1}b \nleqslant q$. We call the pair of elements (a, b) with this property – a *quasi n-zero element* of q. Notice that a zero divisor element of L is a quasi 1-zero element of 0_L, and (a, a, b) is a triple zero element of q if and only if (a, b) is a quasi 2-zero element of q.

Theorem 2.6. *Let q be a weakly quasi n-absorbing element of L. If (a, b) is a quasi n-zero element of q for some $a, b \in L_*$, then $a^n \in ann(q)$ and $b^n \in ann(q)$.*

Proof. Suppose that $a^n \notin ann(q)$. Hence $a^n q_1 \neq 0$ for some $q_1 \in L_*$ where $q_1 \leqslant q$. It follows $0 \neq a^n(b \vee q_1) \leqslant q$. Since $a^n \nleqslant q$, and q is weakly quasi n-absorbing, we conclude that $a^{n-1}(b \vee q_1) \leqslant q$. So $a^{n-1}b \leqslant q$, a contradiction. Thus $a^n q = 0$, and so $a^n \in ann(q)$. Similarly we conclude that $b^n \in ann(q)$. □

Theorem 2.7. *If $\{p_\lambda\}_{\lambda\in\Lambda}$ is a family of (weakly) prime elements of L, then $\bigwedge_{\lambda\in\Lambda} p_\lambda$ is a (weakly) quasi m-absorbing element for all $m \geqslant 2$.*

Proof. Let $\{p_\lambda\}_{\lambda\in\Lambda}$ be a family of prime elements of L. By Corollary 2.3 (3) it is sufficient to prove that $\bigwedge_{\lambda\in\Lambda} p_\lambda$ is a quasi 2-absorbing element of L.

Let $a, b \in L_*$ with $a^2b \leqslant \bigwedge_{\lambda\in\Lambda} p_\lambda$. Since $a^2b \leqslant p_i$ for all prime elements p_i, we have $a \leqslant p_i$ or $b \leqslant p_i$. Thus $ab \leqslant p_i$ for all $i = 1, \ldots, n$ and so $ab \leqslant \bigwedge_{\lambda\in\Lambda} p_\lambda$, which completes the proof for prime elements.

For weakly prime elements the proof is similar. □

Corollary 2.8. *Let q be a proper element of L. Then $\sqrt{q}$, $Nil(L)$ and $J(L)$ are quasi n-absorbing elements of L for all $n \geqslant 2$.*

Proof. It is clear from Theorem 2.7. □

Theorem 2.9. *If $\{q_\lambda\}_{\lambda\in\Lambda}$ is a family of (weakly) quasi m-absorbing elements of a totally ordered lattice L, then for each positive integer m $\bigwedge_{\lambda\in\Lambda} q_\lambda$ is a (weakly) quasi m-absorbing element of L.*

Proof. Assume that $\{q_\lambda\}_{\lambda\in\Lambda}$ is an ascending chain of quasi m-absorbing elements and $a^m \nleqslant \bigwedge_{\lambda\in\Lambda} q_\lambda$ and $a^{m-1}b \nleqslant \bigwedge_{\lambda\in\Lambda} q_\lambda$. We show that $a^m b \nleqslant \bigwedge_{\lambda\in\Lambda} q_\lambda$. Hence $a^m \nleqslant q_j$ and $a^{m-1}b \nleqslant q_k$ for some $j, k = 1, \ldots, n$.

Put $t = \min\{j, k\}$. Then $a^m \nleq q_t$ and $a^{m-1}b \nleq q_t$. Since q_t is a quasi m-absorbing element, it follows $a^m b \nleq q_t$. Thus $a^m b \nleqslant \bigwedge_{\lambda\in\Lambda} q_\lambda$, we are done.

For weakly prime elements the proof is similar. □

Theorem 2.10. *Let for all $i = 1, 2, \ldots, n$, elements $q_1, \ldots, q_n \in L$ are (weakly) quasi m_i-absorbing, respectively. Then $\bigwedge_{i=1}^{n} q_i$ is a (weakly) quasi m-absorbing element of L for $m = \max\{m_1, \ldots, m_n\} + 1$.*

Proof. Suppose that $q_1, \ldots, q_n$ are quasi m_i-absorbing, respectively. Let $a, b \in L_*$ be such that $a^m b \leqslant \bigwedge_{i=1}^{n} q_i$. Hence $a^{m_i} \leqslant q_i$ or $a^{m_i-1}b \leqslant q_i$ for all $i = 1, .., n$. Now assume that $a^m \nleqslant \bigwedge_{i=1}^{n} q_i$. Without loss generality we can suppose that $a^{m_i} \leqslant q_i$ for all $1 \leqslant i \leqslant j$, and $a^{m_i} \nleqslant q_i$ for all $j+1 \leqslant i \leqslant n$. Hence we have $a^{m_i-1}b \leqslant q_i$ for all $j+1 \leqslant i \leqslant n$. Then we get clearly $a^{m-1}b \leqslant q_i$ for $m = \max\{m_1, \ldots, m_n\} + 1$ and for all $1 \leqslant i \leqslant n$. Thus $a^{m-1}b \leqslant \bigwedge_{i=1}^{n} q_i$, so we are done.

For weakly prime elements the proof is similar. □

If $x \in L$, the interval $[x, 1]$ is denoted be L/x. The elemets of $\overline{a}$ and L/x is again a multiplicative lattice with $\overline{a} \circ \overline{b} = ab \vee x$ for all $\overline{a}, \overline{b} \in L/x$.

Theorem 2.11. *Let x and q be proper elements of L with $x \leqslant q$. If q is a (weakly) quasi n-absorbing element of L, then $\overline{q}$ is a (weakly) quasi n-absorbing element of L/x.*

Proof. Suppose that $\overline{a} = a \vee x$, $\overline{b} = b \vee x \in L$ with $\overline{a}^n\overline{b} \leqslant \overline{q}$, where q is a quasi n-absorbing element of L. Then $a^nb \vee x \leqslant q$, and so $a^nb \leqslant q$. Since q is quasi 2-absorbing, we get either $a^n \leqslant q$ or $a^{n-1}b \leqslant q$. Thus $\overline{a}^n = (a \vee x)^n \leqslant \overline{q}$ or $\overline{a}^{n-1}\overline{b} = (a \vee x)^{n-1}(b \vee x) \leqslant \overline{q}$, as needed.

For weakly prime elements the proof is similar. □

Recall that any C-lattice can be localized at a multiplicatively closed set. Let L be a C-lattice and S a multiplicatively closed subset of L_*. Then for $a \in L$, $a_S = \bigvee\{x \in L_* \mid xs \leqslant a \text{ for some } s \in S\}$ and $L_S = \{a_S \mid a \in L\}$. L_S is again a multiplicative lattice under the same order as L with the product $a_S \circ b_s = (a_Sb_S)_S$ where the right hand side is evaluated in L.

If $p \in L$ is prime and $S = \{x \in L_* \mid x \nleqslant p\}$, then L_S is denoted by L_p. [9]

Theorem 2.12. *Let m be a maximal element of L and q be a proper element of L. If q is a (weakly) quasi n-absorbing element of L, then q_m is a (weakly) quasi n-absorbing element of L_m.*

Proof. Let $a, b \in L_*$ such that $a_m^nb_m \leqslant q_m$. Hence $ua^nb \leqslant q$ for some $u \nleqslant m$. It implies that $a^n \leqslant q$ or $a^{n-1}(ub) \leqslant q$. Since $u_m = 1_m$, we get $a_m^n \leqslant q_m$ or $a_m^{n-1}b_m \leqslant q_m$, we are done. □

Theorem 2.13. *Let L be a principal element lattice. Then the following statements are equivalent.*

(1) *Every proper element of L is a quasi n-absorbing element of L.*
(2) *For every a, $b \in L_*$, $a^n = ca^nb$ or $a^{n-1}b = da^nb$ for some c, $d \in L$.*
(3) *For all $a_1, a_2, \ldots, a_{n+1} \in L_*$, $(a_1 \wedge a_2 \wedge \ldots \wedge a_n)^n \leqslant ca_1a_2 \cdots a_{n+1}$ or $(a_1 \wedge a_2 \wedge \ldots \wedge a_n)^{n-1}a_{n+1} \leqslant da_1a_2 \cdots a_{n+1}$ for some c, $d \in L$.*

Proof. (1) $\Leftrightarrow$ (2). Suppose that every proper element of L is a quasi n-absorbing element of L. Hence $a^nb \leqslant (a^nb)$ implies that $a^n \leqslant (a^nb)$ or $a^{n-1}b \leqslant (a^nb)$. Since L is a principal element lattice, there is some element $c \in L$ with $a^n = ca^nb$ or there is some element $d \in L$ with $a^{n-1}b = da^nb$. The converse is clear.

(2) $\Rightarrow$ (3). Put $a = a_1 \wedge a_2 \wedge \ldots \wedge a_n$ and $b = a_{n+1}$. Hence the result follows from (2).

(3) $\Rightarrow$ (2). For all $a, b \in L_*$, we can write $a^n = (\underbrace{a \wedge a \wedge \ldots \wedge a}_{n \text{ times}}) \leqslant ca^nb$ or $a^{n-1}b = (\underbrace{a \wedge a \wedge \ldots \wedge a}_{n-1 \text{ times}})b \leqslant da^nb$. □

Theorem 2.14. *Let $L = L_1 \times L_2$ where L_1 and L_2 are C-lattices. Then:*

(1) *q_1 is a quasi n-absorbing element of L_1 if and only if $(q_1, 1_{L_2})$ is a quasi n-absorbing element of L,*

(2) *q_2 is a quasi n-absorbing element of L_2 if and only if $(1_{L_1}, q_2)$ is a quasi n-absorbing element of L.*

Proof. (1). Suppose that q_1 is a quasi n-absorbing element of L_1.

Let $(a_1, a_2)^n(b_1, b_2) \leqslant (q_1, 1_{L_2})$ for some a_1, $b_1 \in L_{1_*}$ and a_2, $b_2 \in L_{2_*}$. Then $a_1^n b_1 \leqslant q_1$ implies that either $a_1^n \leqslant q_1$ or $a_1^{n-1}b_1 \leqslant q_1$. It follows either $(a_1, a_2)^n \leqslant (q_1, 1_{L_2})$ or $(a_1, a_2)^{n-1}(b_1, b_2) \leqslant (q_1, 1_{L_2})$. Thus $(q_1, 1_{L_2})$ is a quasi n-absorbing element of L. Conversely suppose that $(q_1, 1_{L_2})$ is a quasi n-absorbing element of L and $a^n b \leqslant q_1$ for some $a, b \in L_{1_*}$. Hence $(a, 1_{L_2})^n(b, 1_{L_2}) \leqslant (q_1, 1_{L_2})$ which implies that either $(a, 1_{L_2})^n \leqslant (q_1, 1_{L_2})$ or $(a, 1_{L_2})^{n-1}(b, 1_{L_2}) \leqslant (q_1, 1_{L_2})$. So $a_1^n \leqslant q_1$ or $a_1^{n-1}b_1 \leqslant q_1$, as needed.

(2). It can be verified similar to (1). □

Theorem 2.15. *Let $L = L_1 \times \cdots \times L_k$ where all L_i are C-lattices. If q_i is a quasi n_i-absorbing element of L_i for all $i = 1, \dots, k$, then $(q_1, \dots, q_k)$ is a quasi m-absorbing element of L where $m = \max\{n_1, \dots, n_k\} + 1$.*

Proof. Suppose that $(a_1, \dots, a_k)^m(b_1, \dots, b_k) \leqslant (q_1, \dots, q_k)$ for some $(a_1, \dots, a_k)$, $(b_1, \dots, b_k) \in L_*$ and $m = \max\{n_1, \dots, n_k\} + 1$. Hence $a_i^m b_i = a_i^{n_i}(a_i^{m-n_i}b_i) \leqslant q_i$ for all $i = 1, \dots, k$. Since each q_i is a quasi n_i-absorbing element, we have either $a_i^{n_i} \leqslant q_i$ or $a_i^{m-1}b_i = a_i^{n_i-1}(a_i^{m-n_i}b_i) \leqslant q_i$ for all $i = 1, .., k$. If $a_i^{n_i} \leqslant q_i$ for all $i = 1, \dots, k$, then $(a_1, \dots, a_k)^m \leqslant (q_1, \dots, q_k)$. Without loss generality, suppose that $a_i^{n_i} \leqslant q_i$ for all $1 \leqslant i \leqslant j$ and $a_i^{m-1}b_i \leqslant q_i$ for all $j+1 \leqslant i \leqslant k$, for some $j = 1, \dots, k$. Thus $(a_1, \dots, a_k)^{m-1}(b_1, \dots, b_k) \leqslant (q_1, \dots, q_k)$, so we are done. □

Definition 2.16. A proper element q of L is said to be a *strongly quasi n-absorbing* element of L if whenever $a, b \in L$ (not necessarily compact) with $a^n b \leqslant q$ implies that either $a^n \leqslant q$ or $a^{n-1}b \leqslant q$.

It is clearly seen that every strongly quasi n-absorbing element of L is quasi n-absorbing.

Theorem 2.17. *Let L be a principal element lattice. The following statements are equivalent.*

(1) *Every proper element of L is a strongly quasi n-absorbing element of L.*
(2) *For all $a, b \in L$, $a^n = a^n b$ or $a^{n-1}b = a^n b$.*
(3) *$(a_1 \wedge a_2 \wedge \ldots \wedge a_n)^n \leqslant a_1 a_2 \cdots a_{n+1}$ or $(a_1 \wedge a_2 \wedge \ldots \wedge a_n)^{n-1} a_{n+1} \leqslant a_1 a_2 \cdots a_{n+1}$ for all $a_1, a_2, \dots, a_{n+1} \in L$.*

Proof. This can be easily shown using the similar argument in Theorem 2.13. □

Theorem 2.18. *Let q be a proper element of L. Then:*

(1) *If $a^n b \leqslant q \leqslant a \wedge b$, where $a, b \in L$, implies that $a^n \leqslant q$ or $a^{n-1}b \leqslant q$, then q is a strongly quasi n-absorbing element of L.*
(2) *If $a_1 a_2 \cdots a_{n+1} \leqslant q \leqslant a_1 \wedge a_2 \wedge \ldots \wedge a_{n+1}$, where $a_1, a_2, \dots, a_{n+1} \in L$, implies that $a_1 \cdots a_{i-1} a_{i+1} \cdots a_{n+1} \leqslant q$, for some $1 \leqslant i \leqslant n+1$, then q is a strongly quasi n-absorbing element of L.*

Proof. (1). Let $x, y \in L$ with $x^n y \leqslant q$. We show that $x^n \leqslant q$ or $x^{n-1}y \leqslant q$. Now put $a = x \vee q$ and $b = y \vee q$. Hence we conclude $a^n b \leqslant q \leqslant a \wedge b$, and so $a^n \leqslant q$ or $a^{n-1}b \leqslant q$ by (1). It follows $x^n \leqslant q$ or $x^{n-1}y \leqslant q$.

(2). It can be easily verified similar to (1). □

References

[1] **F. Alarcon and D.D. Anderson**, *Commutative semirings and their lattices of ideals*, Houston J. Math. **20** (1994), 571 – 590.

[2] **D.D. Anderson and C. Jayaram**, *Principal element lattices*, Czechoslovak Math. J. **46** (1996), 99 – 109.

[3] **D.D. Anderson and E. Smith**, *Weakly prime ideals*, Houston J. Math. **29** (2003), 831 – 840.

[4] **D.F. Anderson and A. Badawi**, *On n-absorbing ideals of commutative rings*, Commun. Algebra **39** (2011), 1646 – 1672.

[5] **A. Badawi**, *On 2-absorbing ideals of commutative rings*, Bull. Austral. Math. Soc. **75** (2007), 417 – 429.

[6] **A. Badawi and A.Y. Darani**, *On weakly 2-absorbing ideals of commutative rings*, Houston J. Math. **39** (2013), 441 – 452.

[7] **F. Callialp, C. Jayaram and U. Tekir**, *Weakly prime elements in multiplicative lattices*, Commun. Algebra **40** (2012), 2825 – 2840.

[8] **R.P. Dilworth**, *Abstract commutative ideal theory*, Pacific J. Math. **12** (1962), 481 – 498.

[9] **C. Jayaram and E.W Johnson**, *s-prime elements in multiplicative lattices*, Periodica Math. Hungarica **31** (1995), 201 – 208.

[10] **C. Jayaram, U. Tekir and E. Yetkin**, *2-absorbing and weakly 2-absorbing elements in multiplicative lattices*, Commun. Algebra **42** (2014), 2338 – 2353.

[11] **J.A. Johnson and G.R. Sherette**, *Structural properties of a new class of CM-lattices*, Canadian J. Math. **38** (1986), 552 – 562.

[12] **N.K. Thakare, C.S. Manjarekar and S. Maeda**, *Abstract spectral theory II, Minimal characters and minimal spectrums of multiplicative lattices*, Acta. Sci. Math. (Szeged). **52** (1988), 53 – 67.

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