Quasigroups and Related Systems **24** (2016), 231 − 246

Soft set theoretical approach to residuated lattices

Young Bae Jun and Xiaohong Zhang

Abstract. Molodtsov's soft set theory is applied to residuated lattices. The notion of (filteristic) residuated lattices is introduced, and their properties are investigated. Divisible int-soft filters and strong int-soft filters are defined, and several properties are investigated. Characterizations of a divisible and strong int-soft filter are discussed. Conditions for an int-soft filter to be divisible are established. Relations between a divisible int-soft filter and a strong int-soft filter are considered.

1. Introduction

Various problems in system identification involve characteristics which are essentially non-probabilistic in nature [13]. In response to this situation Zadeh [14] introduced *fuzzy set theory* as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [15]. To solve complicated problem in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [9]. Maji et al. [8] and Molodtsov [9] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [9] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [8] and Çağman et al. [2] described the application of soft set theory to a decision making problem. Maji et al. [7] also studied several operations on the theory of soft sets. Jun and

2010 Mathematics Subject Classification: 03G10, 06B10, 03G25, 06D72.
Keywords: residuated lattice, (filteristic) residuated lattice, (divisible, strong) filter, int-soft filter, divisible int-soft filter, strong int-soft filter.

Park [5] applied the notion of soft sets to BCK/BCI-algebras. In order to deal with fuzzy and uncertain informations, non-classical logic has become a formal and useful tool. As the semantical systems of non-classical logic systems, various logical algebras have been proposed. Residuated lattices are important algebraic structures which are basic of MTL-algebras, BL-algebras, MV-algebras, Gödel algebras, R_0-algebras, lattice implication algebras, etc. The filter theory plays an important role in studying logical systems and the related algebraic structures, and various filters have been proposed in the literature. Zhang et al. [16] introduced the notions of IMTL-filters (NM-filters, MV-filters) of residuated lattices, and presented their characterizations. Ma and Hu [6] introduced divisible filters, strong filters and n-contractive filters in residuated lattices.

In this paper, we apply the notion of soft set theory by Molodtsov to residuated lattices. We introduce the notion of (filteristic) residuated lattices, and investigate their properties. We also define divisible int-soft filters and strong int-soft filters, and investigate related properties. We discuss characterizations of a divisible and strong int-soft filter, and provide conditions for an int-soft filter to be divisible. We establish relations between a divisible int-soft filter and a strong int-soft filter.

2. Preliminaries

We display basic notions on residuated lattices and soft sets which are used in this paper.

Definition 2.1. A *residuated lattice* is an algebra $\mathcal{L} := (L, \vee, \wedge, \otimes, \to, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ such that

(L1) $(L, \vee, \wedge, 0, 1)$ is a bounded lattice,

(L2) $(L, \otimes, 1)$ is a commutative monoid,

(L3) $\otimes$ and $\to$ form an adjoint pair, that is,

$$(\forall x, y, z \in L)\,(x \leqslant y \to z \;\Leftrightarrow\; x \otimes y \leqslant z)\,.$$

In a residuated lattice $\mathcal{L}$, the ordering $\leqslant$ and negation $\neg$ are defined as follows:

$$(\forall x, y \in L)\,(x \leqslant y \;\Leftrightarrow\; x \wedge y = x \;\Leftrightarrow\; x \vee y = y \;\Leftrightarrow\; x \to y = 1)$$

and $\neg x = x \to 0$ for all $x \in L$.

Proposition 2.2 ([1, 3, 4, 6, 11, 12])**.** *In a residuated lattice $\mathcal{L}$, the following properties are valid.*

$$1 \to x = x,\; x \to 1 = 1,\; x \to x = 1,\; 0 \to x = 1,\; x \to (y \to x) = 1, \tag{2.1}$$

$$x \to (y \to z) = (x \otimes y) \to z = y \to (x \to z), \tag{2.2}$$

$$x \leqslant y \;\Rightarrow\; z \to x \leqslant z \to y,\; y \to z \leqslant x \to z, \tag{2.3}$$

$$x \leqslant y \;\Rightarrow\; x \otimes z \leqslant y \otimes z, \tag{2.4}$$

$$z \to y \leqslant (x \to z) \to (x \to y),\; z \to y \leqslant (y \to x) \to (z \to x), \tag{2.5}$$

$$(x \to y) \otimes (y \to z) \leq x \to z. \tag{2.6}$$

$$x \otimes y \leqslant x \otimes (x \to y) \leqslant x \wedge y \leqslant x \wedge (x \to y) \leqslant x, \tag{2.7}$$

$$x \to (y \wedge z) = (x \to y) \wedge (x \to z),\ (x \vee y) \to z = (x \to z) \wedge (y \to z), \tag{2.8}$$

$$x \to y \leqslant (x \otimes z) \to (y \otimes z), \tag{2.9}$$

$$\neg\neg(x \to y) \leqslant \neg\neg x \to \neg\neg y. \tag{2.10}$$

$$\neg x = \neg\neg\neg x,\ x \leqslant \neg\neg x,\ \neg 1 = 0,\ \neg 0 = 1, \tag{2.11}$$

$$x \to (x \wedge y) = x \to y. \tag{2.12}$$

Definition 2.3 ([10]). A nonempty subset F of a residuated lattice $\mathcal{L}$ is called a *filter* of $\mathcal{L}$ if it satisfies the conditions:

$$(\forall x, y \in L)\,(x, y \in F \ \Rightarrow\ x \otimes y \in F)\,, \tag{2.13}$$

$$(\forall x, y \in L)\,(x \in F,\ x \leqslant y \ \Rightarrow\ y \in F)\,. \tag{2.14}$$

Proposition 2.4 ([10]). *A nonempty subset F of a residuated lattice $\mathcal{L}$ is a filter of $\mathcal{L}$ if and only if it satisfies:*

$$1 \in F, \tag{2.15}$$

$$(\forall x \in F)\,(\forall y \in L)\,(x \to y \in F \ \Rightarrow\ y \in F)\,. \tag{2.16}$$

Definition 2.5 ([17]). A soft set $\left(\tilde{f}, L\right)$ over U in a residuated lattice $\mathcal{L}$ is called an *int-soft filter* of $\mathcal{L}$ over U if it satisfies:

$$(\forall x, y \in L)\left(\tilde{f}(x \otimes y) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right), \tag{2.17}$$

$$(\forall x, y \in L)\left(x \leqslant y \ \Rightarrow\ \tilde{f}(x) \subseteq \tilde{f}(y)\right). \tag{2.18}$$

Theorem 2.6 ([17]). *A soft set $\left(\tilde{f}, L\right)$ over U in a residuated lattice $\mathcal{L}$ is an int-soft filter of $\mathcal{L}$ over U if and only if the following assertions are valid:*

$$(\forall x \in L)\left(\tilde{f}(1) \supseteq \tilde{f}(x)\right), \tag{2.19}$$

$$(\forall x, y \in L)\left(\tilde{f}(y) \supseteq \tilde{f}(x \to y) \cap \tilde{f}(x)\right). \tag{2.20}$$

3. (Filteristic) soft residuated lattices

In what follows let $\mathcal{L}$ and A be a residuated lattice and a nonempty set, respectively.

Definition 3.1. Let $(\tilde{f}, A)$ be a soft set over $\mathcal{L}$. Then $(\tilde{f}, A)$ is called a *soft residuated lattice* over $\mathcal{L}$ if $\tilde{f}(x)$ is a sub-residuated lattices of $\mathcal{L}$ for all $x \in A$ with $\tilde{f}(x) \neq \emptyset$. If $\tilde{f}(x)$ is a filter of $\mathcal{L}$ for all $x \in A$ with $\tilde{f}(x) \neq \emptyset$, then $(\tilde{f}, A)$ is called a *filteristic soft residuated lattice* over $\mathcal{L}$.

Example 3.2. Let $L = \{0, a, b, 1\}$ be a chain with the operations $\otimes$ and $\rightarrow$ given by tables

$\otimes$	0	a	b	1
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
1	0	a	b	1

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	0	a	1	1
1	0	a	b	1

Then $\mathcal{L} := (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is a residuated lattice. For $A = \mathbb{N}$, define two soft sets $(\tilde{f}, A)$ and $(\tilde{g}, A)$ over $U = L$ in $\mathcal{L}$ by

$$\tilde{f} : A \rightarrow \mathcal{P}(L), \quad x \mapsto \begin{cases} L & \text{if } x \in \{a \in \mathbb{N} \mid a \leqslant 10\}, \\ \{b, 1\} & \text{otherwise,} \end{cases}$$

and

$$\tilde{g} : A \rightarrow \mathcal{P}(L), \quad x \mapsto \begin{cases} L & \text{if } x \in \{a \in \mathbb{N} \mid a \leqslant 10\}, \\ \{b, 1\} & \text{if } x \in \{a \in \mathbb{N} \mid 10 < a \leqslant 30\}, \\ \{1\} & \text{if } x \in \{a \in \mathbb{N} \mid 30 < a \leqslant 60\}, \\ \emptyset & \text{otherwise,} \end{cases}$$

respectively. Then $(\tilde{f}, A)$ is a soft residuated lattices over $\mathcal{L}$ and $(\tilde{g}, A)$ is a filteristic soft residuated lattice over $\mathcal{L}$.

Theorem 3.3. *Let $(\tilde{f}, A)$ be a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. If B is a subset of A, then $(\tilde{f}|_B, B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$.*

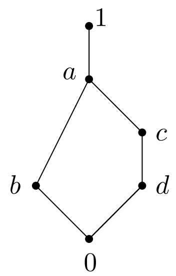
Proof. Straightforward. □

The following example shows that there exists a soft set $(\tilde{f}, A)$ over $\mathcal{L}$ such that

(i) $(\tilde{f}, A)$ is not a soft residuated lattice over $\mathcal{L}$.

(ii) there exists a subset B of A such that $(\tilde{f}|_B, B)$ is a soft residuated lattice over $\mathcal{L}$.

Example 3.4. Consider a residuated lattice $L := \{0, a, b, c, d, 1\}$ with the following Hasse diagram and Cayley tables.



$\otimes$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	a	b	d	d	a
b	c	b	b	0	0	b
c	b	d	0	d	d	c
d	b	d	0	d	d	d
1	0	a	b	c	d	1

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	0	1	b	c	c	1
b	c	a	1	c	c	1
c	b	a	b	1	a	1
d	b	a	b	a	1	1
1	0	a	b	c	d	1

Let $(\tilde{f}, A)$ be a soft set over $\mathcal{L}$, where $A = \mathbb{N}$ and

$$\tilde{f} : A \to \mathcal{P}(L), \quad x \mapsto \begin{cases} L & \text{if } x \in \{a \in \mathbb{N} \mid a \leqslant 10\}, \\ \{a, 1\} & \text{if } x \in \{a \in \mathbb{N} \mid 10 < a \leqslant 20\}, \\ \{b, 1\} & \text{if } x \in \{a \in \mathbb{N} \mid 20 < a \leqslant 30\}, \\ \{c, 1\} & \text{if } x \in \{a \in \mathbb{N} \mid 30 < a \leqslant 40\}, \\ \{d, 1\} & \text{if } x \in \{a \in \mathbb{N} \mid 40 < a \leqslant 50\}, \\ \{c, d, 1\} & \text{otherwise.} \end{cases}$$

Then $(\tilde{f}, A)$ is not a soft residuated lattice over $\mathcal{L}$. But if we take

$$B := \{a \in \mathbb{N} \mid a \leqslant 50\},$$

then $(\tilde{f}|_B, B)$ is a soft residuated lattice over $\mathcal{L}$.

Theorem 3.5. *Let $(\tilde{f}, A)$ and $(\tilde{g}, B)$ be two soft residuated lattices (resp., filteristic soft residuated lattices) over $\mathcal{L}$. If $A \cap B \neq \emptyset$, then the intersection $(\tilde{f}, A)\tilde{\cap}(\tilde{g}, B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$.*

Proof. Note that $(\tilde{f}, A)\tilde{\cap}(\tilde{g}, B) = (\tilde{h}, C)$, where $C = A \cap B$ and $\tilde{h}(x) = \tilde{f}(x)$ or $\tilde{g}(x)$ for all $x \in C$. Note that $\tilde{h} : C \to \mathcal{P}(L)$ is a mapping, and therefore $(\tilde{h}, C)$ is a soft set over $\mathcal{L}$. Since $(\tilde{f}, A)$ and $(\tilde{g}, B)$ are soft residuated lattices (resp., filteristic soft residuated lattices) over $\mathcal{L}$, it follows that $\tilde{h}(x) = \tilde{f}(x)$ is a sub-residuated lattice (resp., filter) of L, or $\tilde{h}(x) = \tilde{g}(x)$ is a sub-residuated lattice (resp., filter) of $\mathcal{L}$ for all $x \in C$. Hence $(\tilde{h}, C) = (\tilde{f}, A)\tilde{\cap}(\tilde{g}, B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. $\square$

Corollary 3.6. *Let $(\tilde{f}, A)$ and $(\tilde{g}, A)$ be two soft residuated lattices (resp., filteristic soft residuated lattices) over $\mathcal{L}$. Then their intersection $(\tilde{f}, A)\tilde{\cap}(\tilde{g}, A)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$.*

Proof. Straightforward. $\square$

Theorem 3.7. *Let $(\tilde{f}, A)$ and $(\tilde{g}, A)$ be two soft residuated lattices (resp., filteristic soft residuated lattices) over $\mathcal{L}$. If A and B are disjoint, then the union $(\tilde{f}, A)\tilde{\cup}(\tilde{g}, A)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$.*

Proof. Note that $(\tilde{f}, A)\widetilde{\cup}(\tilde{g}, B) = (\tilde{h}, C)$, where $C = A \cup B$ and for every $e \in C$,

$$\tilde{h}(e) = \begin{cases} \tilde{f}(e) & \text{if } e \in A \setminus B, \\ \tilde{g}(e) & \text{if } e \in B \setminus A, \\ \tilde{f}(e) \cup \tilde{g}(e) & \text{if } e \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, then $\tilde{h}(x) = \tilde{f}(x)$ is a sub-residuated lattice (resp., filter) of $\mathcal{L}$ since $(\tilde{f}, A)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. If $x \in B \setminus A$, then $\tilde{h}(x) = \tilde{g}(x)$ is a sub-residuated lattice (resp., filter) of $\mathcal{L}$ since $(\tilde{g}, B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. Hence $(\tilde{h}, C) = (\tilde{f}, A)\widetilde{\cup}(\tilde{g}, A)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. □

Theorem 3.8. *If $(\tilde{f}, A)$ and $(\tilde{g}, B)$ are soft residuated lattices (resp., filteristic soft residuated lattices) over $\mathcal{L}$, then $(\tilde{f}, A)\widetilde{\wedge}(\tilde{g}, B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$.*

Proof. Note that $(\tilde{f}, A)\widetilde{\wedge}(\tilde{g}, B) = (\tilde{h}, A \times B)$, where $\tilde{h}(x, y) = \tilde{f}(x) \cap \tilde{g}(y)$ for all $(x, y) \in A \times B$. Since $\tilde{f}(x)$ and $\tilde{g}(y)$ are sub-residuated lattices (resp., filters) of $\mathcal{L}$, the intersection $\tilde{f}(x) \cap \tilde{g}(y)$ is also a sub-residuated lattice (resp., filter) of $\mathcal{L}$. Hence $\tilde{h}(x, y)$ is a sub-residuated lattice (resp., filter) of $\mathcal{L}$ for all $(x, y) \in A \times B$, and therefore $(\tilde{f}, A)\widetilde{\wedge}(\tilde{g}, B) = (\tilde{h}, A \times B)$ is a soft residuated lattice (resp., filteristic soft residuated lattice) over $\mathcal{L}$. □

4. Divisible and strong int-soft filters

Definition 4.1 ([6])**.** A filter F of $\mathcal{L}$ is said to be *divisible* if it satisfies:

$$(\forall x, y \in L)\left((x \wedge y) \to [x \otimes (x \to y)] \in F\right). \tag{4.21}$$

Definition 4.2. An int-soft filter $\left(\tilde{f}, L\right)$ of $\mathcal{L}$ over U is said to be *divisible* if it satisfies:

$$(\forall x, y \in L)\left(\tilde{f}\big((x \wedge y) \to [x \otimes (x \to y)]\big) = \tilde{f}(1)\right). \tag{4.22}$$

Example 4.3. Consider the residuated lattice $\mathcal{L} := (L, \vee, \wedge, \otimes, \to, 0, 1)$ which is given in Example 3.2. Define a soft set $\left(\tilde{f}, L\right)$ over $U = \mathbb{Z}$ in $\mathcal{L}$ by $\tilde{f}(1) = 2\mathbb{Z}$ and $\tilde{f}(x) = 2\mathbb{N}$ for all $x (\neq 1) \in L$. It is routine to verify that $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over $U = \mathbb{Z}$.

Example 4.4. Consider a residuated lattice $L = [0, 1]$ in which two operations "$\otimes$" and "$\to$" are defined as follows:

$$x \otimes y = \begin{cases} 0 & \text{if } x + y \leqslant \frac{1}{2}, \\ x \wedge y & \text{otherwise.} \end{cases}$$

$$x \to y = \begin{cases} 1 & \text{if } x \leqslant y, \\ \left(\frac{1}{2} - x\right) \vee y & \text{otherwise.} \end{cases}$$

The soft set $\left(\tilde{f}, L\right)$ over $U = \mathbb{N}$ in $\mathcal{L}$ given by $\tilde{f}(1) = 3\mathbb{N}$ and $\tilde{f}(x) = 6\mathbb{N}$ for all $x(\neq 1) \in L$ is an int-soft filter of $\mathcal{L}$. But it is not divisible since

$$\tilde{f}((0.3 \wedge 0.2) \to (0.3 \otimes (0.3 \to 0.2)) = \tilde{f}(0.3) \neq \tilde{f}(1).$$

Proposition 4.5. *Every divisible int-soft filter $\left(\tilde{f}, L\right)$ of $\mathcal{L}$ over U satisfies the following identity.*

$$\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z))) = \tilde{f}(1) \tag{4.23}$$

for all $x, y, z \in L$.

Proof. Let $x, y, z \in L$. If we let $x := x \otimes y$ and $y := x \otimes z$ in (4.22), then

$$\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to ((x \otimes y) \otimes ((x \otimes y) \to (x \otimes z)))) = \tilde{f}(1). \tag{4.24}$$

Using (2.2) and (2.7), we have

$$\begin{aligned}(x \otimes y) \otimes ((x \otimes y) \to (x \otimes z)) &= x \otimes y \otimes (y \to (x \to (x \otimes z))) \\ &\leqslant x \otimes (y \wedge (x \to (x \otimes z))),\end{aligned}$$

which implies from (2.3)

$$\begin{aligned}&((x \otimes y) \wedge (x \otimes z)) \to ((x \otimes y) \otimes ((x \otimes y) \to (x \otimes z))) \\ &\leqslant ((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))).\end{aligned}$$

It follows from (4.24) and (2.18) that

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to ((x \otimes y) \otimes ((x \otimes y) \to (x \otimes z)))) \\ &\subseteq \tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))))\end{aligned}$$

and so that

$$\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z))))) = \tilde{f}(1). \tag{4.25}$$

On the other hand, if we take $x := x \to (x \otimes z)$ in (4.22) then

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}((y \wedge (x \to (x \otimes z))) \to ((x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ &\subseteq \tilde{f}((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &\qquad (x \otimes ((x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ &= \tilde{f}((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &\qquad (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))\end{aligned}$$

by using (2.9), (2.18) and the commutativity and associativity of $\otimes$. Hence

$$\tilde{f}(1) = \tilde{f}((x \otimes (y \wedge (x \to (x \otimes z)))) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))). \tag{4.26}$$

Using (2.6), we obtain

$$\begin{aligned}&(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))))\otimes \\ &\quad ((x \otimes (y \wedge (x \to (x \otimes z)))) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ &\leqslant ((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)).\end{aligned}$$

It follows from (2.18), (2.17), (4.25) and (4.26) that

$$\begin{aligned}&\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ &\supseteq \tilde{f}((((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))))\otimes \\ &\quad ((x \otimes (y \wedge (x \to (x \otimes z)))) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ &\supseteq \tilde{f}((((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z))))))\cap \\ &\qquad \tilde{f}(((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &\qquad\qquad (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ &= \tilde{f}(1)\end{aligned}$$

Thus

$$\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) = \tilde{f}(1). \tag{4.27}$$

Since

$$x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)) \leqslant x \otimes z \otimes (z \to y) \leqslant x \otimes (y \wedge z),$$

we get

$$\begin{aligned}&((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ &\leqslant ((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z)).\end{aligned}$$

It follows that

$$\begin{aligned}&\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z))) \\ &\supseteq \tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ &= \tilde{f}(1)\end{aligned}$$

and that $\tilde{f}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z))) = \tilde{f}(1)$. □

We consider characterizations of a divisible int-soft filter.

Theorem 4.6. *An int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *is divisible if and only if the following assertion is valid:*

$$\tilde{f}\big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\big) = \tilde{f}(1) \tag{4.28}$$

for all $x, y, z \in L$.

Proof. Assume that $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U. If we take $x := x \to y$ and $y := x \to z$ in (4.22) and use (2.8) and (2.2), then

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}\left([(x \to y) \wedge (x \to z)] \to [(x \to y) \otimes ((x \to y) \to (x \to z))]\right)\\ &= \tilde{f}\left([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\right).\end{aligned}$$

Using (2.5) and (2.9), we have

$$\begin{aligned}&(x \wedge y) \to [x \otimes (x \to y)] \leqslant [(x \otimes (x \to y)) \to z] \to [(x \wedge y) \to z]\\ &\leqslant [(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\end{aligned}$$

for all $x, y, z \in L$. Since $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U, it follows from (4.22) and (2.18) that

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}((x \wedge y) \to [x \otimes (x \to y)])\\ &\subseteq \tilde{f}([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)])\end{aligned}$$

and so from (2.19) that

$$\tilde{f}([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]) = \tilde{f}(1)$$

for all $x, y, z \in L$. Using (2.6), we get

$$\begin{aligned}&\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\Big) \otimes\\ &\qquad\Big([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big)\\ &\leqslant [x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)],\end{aligned}$$

and so

$$\begin{aligned}&\tilde{f}\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big)\\ &\supseteq \tilde{f}\Big(([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]) \otimes\\ &\qquad([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)])\Big)\\ &\supseteq \tilde{f}\big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\big) \cap\\ &\qquad\tilde{f}\big([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\big)\\ &= \tilde{f}(1).\end{aligned}$$

Therefore

$$\tilde{f}\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big) = \tilde{f}(1)$$

for all $x, y, z \in L$.

Conversely, let $\left(\tilde{f}, L\right)$ be an int-soft filter that satisfies the condition (4.28). If we take $x := 1$ in (4.28) and use (2.1), then we obtain (4.22). □

Theorem 4.7. *An int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *is divisible if and only if it satisfies:*

$$\tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right) = \tilde{f}(1) \tag{4.29}$$

for all $x, y \in L$.

Proof. Suppose that $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U. Note that

$$(x \wedge y) \to [x \otimes (x \to y)] \leq [y \otimes (y \to x)] \to [x \otimes (x \to y)]$$

for all $x, y \in L$. It follows from (4.22) and (2.18) that

$$\tilde{f}(1) = \tilde{f}\left((x \wedge y) \to [x \otimes (x \to y)]\right) \subseteq \tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right)$$

and that $\tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right) = \tilde{f}(1)$.

Conversely, let $\left(\tilde{f}, L\right)$ be an int-soft filter of $\mathcal{L}$ over U that satisfies the condition (4.29). Since $y \to x = y \to (y \wedge x)$ for all $x, y \in L$, the condition (4.29) implies that

$$\tilde{f}\left([y \otimes (y \to (x \wedge y))] \to [x \otimes (x \to (x \wedge y))]\right) = \tilde{f}(1). \tag{4.30}$$

If we take $y := x \wedge z$ in (4.30), then

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}\left([(x \wedge z) \otimes ((x \wedge z) \to (x \wedge (x \wedge z)))] \to [x \otimes (x \to (x \wedge (x \wedge z)))]\right)\\ &= \tilde{f}\left((x \wedge z) \to [x \otimes (x \to z)]\right).\end{aligned}$$

Therefore $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U. □

We discuss conditions for an int-soft filter to be divisible.

Theorem 4.8. *If an int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *satisfies the following assertion:*

$$\tilde{f}((x \wedge y) \to (x \otimes y)) = \tilde{f}(1) \tag{4.31}$$

for all $x, y \in L$*, then* $\left(\tilde{f}, L\right)$ *is divisible.*

Proof. Note that $x \otimes y \leqslant x \otimes (x \to y)$ for all $x, y \in L$. It follows from (2.3) that

$$(x \wedge y) \to (x \otimes y)] \leqslant (x \wedge y) \to (x \otimes (x \to y)).$$

Hence, by (4.31) and (2.18), we have

$$\tilde{f}(1) = \tilde{f}((x \wedge y) \to (x \otimes y)) \subseteq \tilde{f}((x \wedge y) \to (x \otimes (x \to y))),$$

and so $\tilde{f}((x \wedge y) \to (x \otimes (x \to y))) = \tilde{f}(1)$ for all $x, y \in L$. Therefore $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U. □

Theorem 4.9. *If an int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *satisfies the following assertion:*

$$\tilde{f}((x \wedge (x \to y)) \to y) = \tilde{f}(1) \tag{4.32}$$

for all $x, y \in L$ *then* $\left(\tilde{f}, L\right)$ *is divisible.*

Proof. If we take $y := x \otimes y$ in (4.32), then

$$\tilde{f}(1) = \tilde{f}((x \wedge (x \to (x \otimes y))) \to (x \otimes y)) \subseteq \tilde{f}((x \wedge y) \to (x \otimes y))$$

and so $\tilde{f}((x \wedge y) \to (x \otimes y)) = \tilde{f}(1)$ for all $x, y \in L$. It follows from Theorem 4.8 that $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U. □

Theorem 4.10. *If an int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *satisfies the following assertion:*

$$\tilde{f}(x \to z) \supseteq \tilde{f}((x \otimes y) \to z) \cap \tilde{f}(x \to y) \tag{4.33}$$

for all $x, y, z \in L$, *then* $\left(\tilde{f}, L\right)$ *is divisible.*

Proof. If we take $x := x \wedge (x \to y)$, $y := x$ and $z := y$ in (4.33), then

$$\begin{aligned}
&\tilde{f}((x \wedge (x \to y)) \to y) \\
&\supseteq \tilde{f}(((x \wedge (x \to y)) \otimes x) \to y) \cap \tilde{f}((x \wedge (x \to y)) \to x) \\
&= \tilde{f}(1).
\end{aligned}$$

Thus $\tilde{f}((x \wedge (x \to y)) \to y) = \tilde{f}(1)$ for all $x, y \in L$, and so $\left(\tilde{f}, L\right)$ is a divisible int-soft filter of $\mathcal{L}$ over U by Theorem 4.9. □

Theorem 4.11. *If an int-soft filter* $\tilde{f}$ *of* $\mathcal{L}$ *over* U *satisfies the following assertion:*

$$\tilde{f}(x \to (x \otimes x)) = \tilde{f}(1) \tag{4.34}$$

for all $x \in L$, *then* $\left(\tilde{f}, L\right)$ *is divisible.*

Proof. Let $\left(\tilde{f}, L\right)$ be an int-soft filter of $\mathcal{L}$ over U that satisfies the condition (4.34). Using (2.9) and the commutativity of $\otimes$, we have

$$x \to y \leqslant (x \otimes x) \to (x \otimes y),$$

and so

$$(x \to (x \otimes x)) \otimes (x \to y) \leqslant (x \to (x \otimes x)) \otimes ((x \otimes x) \to (x \otimes y))$$

for all $x, y \in L$ by (2.4) and the commutativity of $\otimes$. It follows from (2.6), (2.4) and the commutativity of $\otimes$ that

$$\begin{aligned}
&((x \to (x \otimes x)) \otimes (x \to y)) \otimes ((x \otimes y) \to z) \\
&\leqslant ((x \to (x \otimes x)) \otimes ((x \otimes x) \to (x \otimes y))) \otimes ((x \otimes y) \to z) \\
&\leqslant (x \to (x \otimes y)) \otimes ((x \otimes y) \to z) \\
&\leqslant x \to z
\end{aligned}$$

and so from (2.17), (2.18), (2.19) and (4.34) that

$$\begin{aligned}
\tilde{f}(x \to z) &\supseteq \tilde{f}(((x \to (x \otimes x)) \otimes (x \to y)) \otimes ((x \otimes y) \to z)) \\
&\supseteq \tilde{f}((x \to (x \otimes x)) \otimes (x \to y)) \cap \tilde{f}((x \otimes y) \to z) \\
&\supseteq \tilde{f}(x \to (x \otimes x)) \cap \tilde{f}(x \to y) \cap \tilde{f}((x \otimes y) \to z) \\
&= \tilde{f}(1) \cap \tilde{f}(x \to y) \cap \tilde{f}((x \otimes y) \to z) \\
&= \tilde{f}((x \otimes y) \to z) \cap \tilde{f}(x \to y)
\end{aligned}$$

for all $x, y, z \in L$. Therefore $\left(\tilde{f}, L\right)$ is a divisible int-soft fillter of $\mathcal{L}$ over U by Theorem 4.10. □

Definition 4.12 ([6]). A filter F of $\mathcal{L}$ is said to be *strong* if it satisfies:

$$\neg\neg(\neg\neg x \to x) \in F \tag{4.35}$$

for all $x \in L$.

Definition 4.13. An int-soft filter $\left(\tilde{f}, L\right)$ of $\mathcal{L}$ over U is said to be *strong* if it satisfies:

$$\tilde{f}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{f}(1) \tag{4.36}$$

for all $x \in L$.

Example 4.14. Consider the residuated lattice $\mathcal{L} := (L, \vee, \wedge, \otimes, \to, 0, 1)$ which is given in Example 3.4. Define a soft set $\left(\tilde{f}, L\right)$ over $U = \mathbb{Z}$ in $\mathcal{L}$ by $\tilde{f}(1) = 3\mathbb{Z}$ and $\tilde{f}(x) = 6\mathbb{N}$ for all $x(\neq 1) \in L$. It is routine to check that $\left(\tilde{f}, L\right)$ is a strong int-soft filter of $\mathcal{L}$ over $U = \mathbb{Z}$.

We provide characterizations of a strong int-soft filter.

Theorem 4.15. *Given a soft set* $\left(\tilde{f}, L\right)$ *over* U *in* $\mathcal{L}$*, the following assertions are equivalent.*

(i) $\left(\tilde{f}, L\right)$ *is a strong int-soft filter of* $\mathcal{L}$ *over* U*.*

(ii) $\left(\tilde{f}, L\right)$ *is an int-soft filter of* $\mathcal{L}$ *over* U *that satisfies*

$$(\forall x, y \in L)\left(\tilde{f}((y \to \neg\neg x) \to \neg\neg(y \to x) = \tilde{f}(1)\right). \tag{4.37}$$

(iii) $\left(\tilde{f}, L\right)$ *is an int-soft filter of* $\mathcal{L}$ *over* U *that satisfies*

$$(\forall x, y \in L)\left(\tilde{f}((\neg x \to y) \to \neg\neg(\neg y \to x)) = \tilde{f}(1)\right). \tag{4.38}$$

Proof. Assume that $\left(\tilde{f}, L\right)$ is a strong int-soft filter of $\mathcal{L}$ over U. Then $\left(\tilde{f}, L\right)$ is an int-soft filter of $\mathcal{L}$ over U. Note that

$$\begin{aligned}\neg\neg(\neg\neg x \to x) &\leqslant \neg\neg((y \to \neg\neg x) \to (y \to x))\\ &\leqslant \neg\neg((y \to \neg\neg x) \to \neg\neg(y \to x))\\ &= (y \to \neg\neg x) \to \neg\neg(y \to x)\end{aligned}$$

and

$$\begin{aligned}\neg\neg(\neg\neg x \to x) &\leqslant \neg\neg(((\neg x \to y) \otimes \neg y) \to x)\\ &= \neg\neg((\neg x \to y) \to (\neg y \to x))\\ &\leqslant \neg\neg((\neg x \to y) \to \neg\neg(\neg y \to x))\\ &= (\neg x \to y) \to \neg\neg(\neg y \to x)\end{aligned}$$

for all $x, y \in L$. If follows from (4.36) and (2.18) that

$$\tilde{f}(1) = \tilde{f}(\neg\neg(\neg\neg x \to x)) \subseteq \tilde{f}((y \to \neg\neg x) \to \neg\neg(y \to x)) \tag{4.39}$$

and

$$\tilde{f}(1) = \tilde{f}(\neg\neg(\neg\neg x \to x)) \subseteq \tilde{f}((\neg x \to y) \to \neg\neg(\neg y \to x)). \tag{4.40}$$

Combining (2.19), (4.39) and (4.40), we have $\tilde{f}((y \to \neg\neg x) \to \neg\neg(y \to x)) = \tilde{f}(1)$ and $\tilde{f}((\neg x \to y) \to \neg\neg(\neg y \to x)) = \tilde{f}(1)$ for all $x, y \in L$. Therefore (ii) and (iii) are valid. Let $\left(\tilde{f}, L\right)$ be an int-soft filter of $\mathcal{L}$ over U that satisfies the condition (4.37). If we take $y := \neg\neg x$ in (4.37) and use (2.1), then we can induce the condition (4.36) and so $\left(\tilde{f}, L\right)$ is a strong int-soft filter of $\mathcal{L}$ over U. Let $\left(\tilde{f}, L\right)$ be an int-soft filter of $\mathcal{L}$ over U that satisfies the condition (4.38). Taking $y := \neg x$ in (4.38) and using (2.1) induces the condition (4.36). Hence $\left(\tilde{f}, L\right)$ is a strong int-soft filter of $\mathcal{L}$ over U. □

We investigate relationship between a divisible int-soft filter and a strong int-soft filter.

Theorem 4.16. *Every divisible int-soft filter is a strong int-soft filter.*

Proof. Let $\left(\tilde{f}, L\right)$ be a divisible int-soft filter of $\mathcal{L}$ over U. If we put $x := \neg\neg x$ and $y := x$ in (4.22), then we have

$$\tilde{f}((\neg\neg x \wedge x) \to (\neg\neg x \otimes (\neg\neg x \to x))) = \tilde{f}(1). \tag{4.41}$$

Using (2.5) and (2.4), we get

$$\begin{aligned}
&(\neg\neg x \wedge x) \to (\neg\neg x \otimes (\neg\neg x \to x)) \leqslant \neg(\neg\neg x \otimes (\neg\neg x \to x)) \to \neg(\neg\neg x \wedge x)\\
&\leqslant (\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x))) \to (\neg\neg x \otimes \neg(\neg\neg x \wedge x))\\
&\leqslant \neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))
\end{aligned}$$

for all $x \in L$. It follows from (4.41) and (2.18) that

$$\begin{aligned}
\tilde{f}(1) &= \tilde{f}((\neg\neg x \wedge x) \to (\neg\neg \otimes (\neg\neg x \to x)))\\
&\subseteq \tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))).
\end{aligned} \tag{4.42}$$

Combining (4.42) with (2.19), we have

$$\tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) = \tilde{f}(1) \tag{4.43}$$

for all $x \in L$. Using (2.2), (2.10), (2.12) and (2.11), we get

$$\begin{aligned}
\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) &= \neg\neg x \to \neg\neg(\neg\neg x \wedge x)\\
&\geq \neg\neg(x \to (\neg\neg x \wedge x))\\
&= \neg\neg(x \to (x \wedge \neg\neg x))\\
&= \neg\neg(x \to \neg\neg x) = \neg\neg 1 = 1
\end{aligned}$$

and so $\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) = 1$ for all $x \in L$. It follows from (4.43) and (2.20) that

$$\begin{aligned}
&\tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x))))\\
&\supseteq \tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) \cap\\
&\qquad\qquad \tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)))\\
&= \tilde{f}(1)
\end{aligned}$$

and so that

$$\begin{aligned}
\tilde{f}(1) &= \tilde{f}(\neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x))))\\
&= \tilde{f}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x)))).
\end{aligned} \tag{4.44}$$

Taking $x := \neg\neg x$ and $y := \neg(\neg\neg x \to x)$ in (4.22) induces

$$\begin{aligned}\tilde{f}(1) &= \tilde{f}((\neg\neg x \wedge \neg(\neg\neg x \to x)) \to (\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))))\\ &\subseteq \tilde{f}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))) \to \neg(\neg\neg x \wedge \neg(\neg\neg x \to x)))\end{aligned}$$

by using (2.3) and (2.18). Thus

$$\tilde{f}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))) \to \neg(\neg\neg x \wedge \neg(\neg\neg x \to x))) = \tilde{f}(1). \quad (4.45)$$

Since $\neg(\neg\neg x \to x) \leqslant \neg\neg x$ for all $x \in L$, it follows from (2.19), (2.20), (4.44) and (4.45) that

$$\tilde{f}(1) = \tilde{f}(\neg(\neg\neg x \wedge \neg(\neg\neg x \to x))) = \tilde{f}(\neg\neg(\neg\neg x \to x))$$

for all $x \in L$. Therefore $\left(\tilde{f}, L\right)$ is a strong int-soft filter of $\mathcal{L}$ over U. □

Corollary 4.17. *If an int-soft filter* $\left(\tilde{f}, L\right)$ *of* $\mathcal{L}$ *over* U *satisfies one of conditions* (4.28)*,* (4.29)*,* (4.31)*,* (4.32)*,* (4.33) *and* (4.34)*, then* $\tilde{f}$ *is a strong int-soft filter of* $\mathcal{L}$ *over* U*.*

The following example shows that the converse of Theorem 4.16 may not be true in general.

Example 4.18. The strong int-soft filter $\tilde{f}$ of $\mathcal{L}$ over U which is given in Example 4.14 is not a divisible int-soft filter of $\mathcal{L}$ over U since

$$\tilde{f}((a \wedge c) \to (a \otimes (a \to c))) = \tilde{f}(a) \neq \tilde{f}(1).$$

5 Conclusions

We have considered the soft set theoretical approach to residuated lattices. We have discussed (filteristic) soft residuated lattices We have defined divisible int-soft filters and strong int-soft filters, and have investigated related properties. We have discussed characterizations of a divisible and strong int-soft filter, and have provided conditions for an int-soft filter to be divisible. We have establish relations between a divisible int-soft filter and a strong int-soft filter. In a forthcoming paper, we will study the int-soft version of n-contractive filters in residuated lattices, and apply the results to the another type filters in residuated lattices.

Acknoweledgments. This work was supported by National Natural Science Foundation of China (Grant No. 61573240, 61473239).

References

[1] **R. Belohlavek**, *Some properties of residuated lattices,* Czechoslovak Math. J. **53(123)** (2003), 161 – 171.

[2] **N. Çağman and S. Enginoğlu**, *Soft set theory and uni-int decision making,* Eur. J. Oper. Res. **207** (2010), 848 – 855.

[3] **F. Esteva and L. Godo**, *Monoidal t-norm based logic, towards a logic for left-continuous t-norms,* Fuzzy Sets and Systems **124** (2001), 271 – 288.

[4] **P. Hájek**, *Metamathematics of Fuzzy Logic,* Kluwer Academic Press, Dordrecht, 1998.

[5] **Y.B. Jun and C.H. Park**, *Applications of soft sets in ideal theory of BCK/BCI-algebras,* Inform. Sci. **178** (2008), 2466 – 2475.

[6] **Z.M. Ma and B.Q. Hu**, *Characterizations and new subclasses of $\mathcal{I}$-filters in residuated lattices,* Fuzzy Sets and Systems **247** (2014), 92 – 107.

[7] **P.K. Maji, R. Biswas and A.R. Roy**, *Soft set theory,* Comput. Math. Appl. **45** (2003), 555 – 562.

[8] **P.K. Maji, A.R. Roy and R. Biswas**, *An application of soft sets in a decision making problem,* Comput. Math. Appl. **44** (2002), 1077 – 1083.

[9] **D. Molodtsov**, *Soft set theory - First results,* Comput. Math. Appl. **37** (1999), 19 – 31.

[10] **J.G. Shen and X.H. Zhang**, *Filters of residuated lattices.* Chin. Quart. J. Math. **21** (2006), 443 – 447.

[11] **E. Turunen**, *BL-algebras of basic fuzzy logic,* Mathware Soft Comput. **6** (1999), 49 – 61.

[12] **E. Turunen**, *Boolean deductive systems of BL-algebras,* Arch. Math. Logic **40** (2001), 467 – 473.

[13] **L.A. Zadeh**, *From circuit theory to system theory,* Proc. Inst. Radio Eng. **50** (1962), 856 – 865.

[14] **L.A. Zadeh**, *Fuzzy sets,* Information and Control **8** (1965), 338–353.

[15] **L.A. Zadeh**, *Toward a generalized theory of uncertainty (GTU) – an outline,* Inform. Sci. **172** (2005), 1 – 40.

[16] **X.H. Zhang, H. Zhou and X. Mao**, *$IMTL(MV)$-filters and fuzzy $IMTL(MV)$-filters of residuated lattices,* J. Intell. Fuzzy Systems **26** (2014), 589 – 596.

[17] **Y.Q. Zhu and Y. Xu**, *On filter theory of residuated lattices,* Inform. Sci. **180** (2010), 3614 – 3632.

Rceived March 2, 2016

Y.B. Jun
Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea
E-mail: skywine@gmail.com

X.H. Zhang
Department of Mathematics, College of Arts and Sciences, Shanghai Maritime University,
Shanghai 201306, P.R. China
E-mail: zxhonghz@263.net