Quasigroups and Related Systems **24** (2016), 247 − 248

On the paper "On dual ordered semigroups"

Niovi Kehayopulu

Abstract. This is about the paper by Thawhat Changphas and Nawamin Phaipong in Quasigroups and Related Systems 22 (2014), 193–200.

This paper is actually the introduction and the main part from section 2 (the first decomposition theorem) of the paper by St. Schwarz [3]. The authors considered an ordered semigroup $(S, \cdot, \leq)$ instead of the semigroup $(S, \cdot)$ considered by Schwarz. The order plays a very little role in Lemma 1.1(1), Lemma 1.1(4), Lemma 2.7 and Lemma 2.15(3),(4) (the Lemma 2.8 and Corollaries 2.9 and 2.10 are immediate consequences of Lemma 2.7), but the proofs of Lemma 1.1(1) and Lemma 1.1(4) are wrong. In addition, the Lemma 1.1(4) does not need any proof since it is an immediate consequence of Lemma 1.1(1). This is a copy of the proof of the Lemma 1.1(1) in [1]: *If* $x \in l(A)$ *and* $S \ni y \leq x$*, then* $yA \subseteq xA = 0$*, and hence* $y \in l(A)$. As we see, according to this proof, we first have $yA \subseteq xA$ (there is no the proof in the paper) and then, based on it, we have $y \in l(A)$. But to prove that $x \in l(A)$ and $S \ni y \leq x$ implies $yA \subseteq xA$ (that actually implies $yA = xA$), we first have to prove that $y \in l(A)$. Then our argument is finished and we do not go back to use the $yA \subseteq xA$ to prove that $y \in l(A)$ (which has been already proved before the proof of the $yA \subseteq xA$). So when the authors say "If $x \in l(A)$ and $S \ni y \leq x$ implies $yA \subseteq xA$", they cannot mean anything else than the "$y \leq x$ implies $yA \subseteq xA$", and this is not true in general. The same problem occurs in the proof of Lemma 1.1(4) of the paper in [1]. It might be mentioned here that $y \leq x$ implies $yA \subseteq (xA]$.

Let us prove that if M is a right (resp. left) ideal of an ordered semigroup S, then $y \leq x$ does not imply $My \subseteq Mx$ (resp. $yM \subseteq xM$) in general. This shows the mistake in Lemma 1.1(1) as well, as the right and the left ideals of an ordered semigroup S are nonempty subsets of S.

Example. [2] Consider the ordered semigroup $S = \{a, b, c, d, f\}$ defined by the multiplication and the covering relation given below:

2010 Mathematics Subject Classification: 06F05, 20M10
Keywords: dual semigroup; dual ordered semigroup; left (right) ideal.

$\cdot$	a	b	c	d	f
a	a	b	a	a	a
b	a	b	a	a	a
c	a	b	c	a	a
d	a	b	a	a	d
f	a	b	a	a	f

$$\prec = \{a,b),(c,a),(d,a)\}.$$

The set $M = \{a,c,d\}$ is a left ideal of S, $c \le a$ but $cM \nsubseteq aM$. The set $M = \{a,b,c,d\}$ is a right ideal of S, $c \le a$ but $Mc \nsubseteq Ma$. $\square$

This is the corrected form of Lemma 1.1(1) and its proof:

Lemma 1.1(1). *If $(S,.,\le)$ is an ordered semigroup with zero and A a nonempty subset of S, then the set $l(A)$ is a left ideal and the set $r(A)$ is a right ideal of S.*

Proof. The set $l(A)$ is a left ideal of the semigroup $(S,.)$ [3]. Let now $x \in l(A)$ and $S \ni y \le x$. Then $y \in l(A)$, that is, $yA = \{0\}$. Indeed: if $z \in A$, then $yz \le xz \in xA = \{0\}$, so $yz = 0$. Since $yA \subseteq \{0\}$ and $yA \neq \emptyset$, we have $yA = \{0\}$. $\square$

References

[1] **T. Changphas and N. Phaipong**, *On dual ordered semigroups*, Quasigroups and Related Systems **22** (2014), 193 − 200.

[2] **N. Kehayopulu**, *On regular, intra-regular ordered semigroups*, Pure Math. Appl. (PU.M.A.) **4** (1993), 447 − 461.

[3] **S. Schwarz**, *On dual semigroups*, Czechoslovak Math. J. **10** (85) (1960), 201 − 230.

Received May 12, 2016

University of Athens
Department of Mathematics
15784 Panepistimiopolis
Athens, Greece
E-mail: nkehayop@math.uoa.gr