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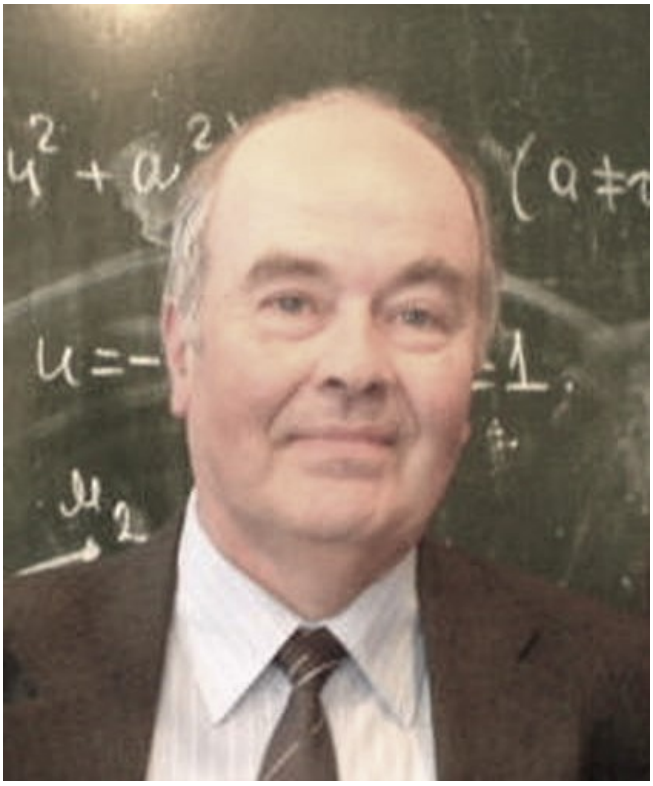
In memoriam: Valentin S. Trokhimenko



Valentin S. Trokhimenko (15.02.1941–21.11.2020)

Valentin Stepanovich Trokhimenko was born on February 15, 1941 in the village Shpikov near Vinnitsa. Parents came from peasants, but despite the difficulties of that historical period, they were able to get a higher education. During the Second World War, his father was a war correspondent, after the war he was engaged in administrative work. Mother was a mathematics teacher in primary school.

After the war, the Trochimenko family moved to Vinnitsa. In 1963, Valentin graduated from the Vinnitsa Pedagogical Institute and became an assistant at the Faculty of Mathematics. In 1967, he entered the postgraduate school of V.V. Vagner at the Saratov State University. In 1970 he was sent to work as an assistant

at the Faculty of Higher Mathematics of the Ulyanovsk Polytechnic Institute, where he worked for 3 years as a senior lecturer. In 1973, V.S. Trokhimenko returned to Vinnitsa and continued his work at the Faculty of Mathematics of the Vinnitsa Pedagogical University, where he worked for over 40 years.

Valentin drew well (pencil graphics), had excellent hearing, played the violin and flute, was fond of the history of music, was interested in astronomy and physics. He loved people very much, he loved communication with students and colleagues, and they paid him the same. He was a gifted teacher and an excellent lecturer liked by students.

From the very beginning of his scientific activity, V.S. Trochimenko studied algebras of functions of several variables. Only a few articles (at the beginning of his activity) were devoted to semigroups.

On the set Φ of functions $G^n \to G$ we have defined an $(n+1)$-ary operation $\mathcal{O}(g, f_1, \ldots, f_n) = g[f_1, \ldots, f_n]$, where $g[f_1, \ldots, f_n](\mathbf{x}) = g(f_1(\mathbf{x}), \ldots, f_n(\mathbf{x}))$ for $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in G^n$. It is called the Menger's superposition. Such defined operation $\mathcal{O}$ is superassociative, i.e.

$$\mathcal{O}(\mathcal{O}(g, f_1, f_2, \ldots, f_n), h_1^n) = \mathcal{O}(g, \mathcal{O}(f_1, h_1^n), \mathcal{O}(f_2, h_1^n), \ldots, \mathcal{O}(f_2, h_1^n))$$

holds for all $f_i, h_j \in \Phi$, where $h_1^n = h_1, h_2, \ldots, h_n$. The set Φ of n-place functions closed under such defined superposition is called the Menger algebra of n-place functions. For $n = 1$ we obtain ordinary semigroups.

In the theory of n-place functions also are considered the so-called Mann's compositions:

$$(f \underset{i}{\oplus} g)(x_1, x_2, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, g(x_1, x_2, \ldots, x_n), x_{i+1}, \ldots, x_n).$$

We have n such compositions: $\underset{1}{\oplus}, \underset{2}{\oplus}, \ldots, \underset{n}{\oplus}$. Each of these compositions is an associative operation. Thus the set of n-place functions closed with respect to these compositions is called an $(2, n)$-semigroup of functions. If it also is closed with respect to the Menger's composition, then it is called the Menger $(2, n)$-semigroup.

The systematic study of such algebras was initiated in the mid-1940s by K. Menger. Twenty years later, B.M. Schein (inspired by V.V. Vagner) began to study abstract algebras isomorphic to algebras of single or multiplace functions, creating a group of young collaborators in Saratov. Among them was V.S. Trokhimenko who prepared (under the supervision of B.M. Schein) the PhD dissertation "Algebras of multiplace functions" (1970).

The first results of the V.S. Trochimenko were published (in Russian) in the publications of the University of Saratov and concerned the characterization of certain classes of functions with $n > 1$ variables defined on a given set. They were classes of functions with given properties, for example, classes of invertible functions. Later, his attention was drawn to various dependencies between these functions, which led to the study of Menger algebras with given partial order. We

should mention the obtained characterizations of algebras of partial functions with disjoint domains or the characterizations of algebras of functions with intersecting domains. His indisputable merit was showing how important a role setectors play in studying these functions.

I met Valentin in the mid-1990s at a conference in Vinnitsa, where I had the opportunity to listen to his interesting plenary talk on unsolved problems related to Menger algebras. Since then, we started to correspond and exchange the results. This resulted in over a dozen articles and a monograph, first published (in Russian) and then in a much enlarged version in English (in 2012). In this monograph we have included many unknown, but interesting, results of many Russian authors published in Russian-language publications of local universities.

In the meantime, we noticed that in the study of multiplace functions algebras, in addition to Menger's superposition, Mann's superpositions also play an important role. Menger $(2, n)$-semigroups of multiplace functions has a much richer structure. It also allows for a number of relations that facilitate the study of algebras of functions.

In recent years, we have been looking for the characterization of algebras of functions that allow a certain fixed permutation of variables. Here Valentin had a great intuition in searching for a minimal system of axioms defining individual classes of algebras of functions. He also had many ideas for further research. He continued them almost until the end of his life. Unfortunately, his illness prevented him from making a note of the virus COVID-19 ended his life. I am concerned that many of his unpublished results have been lost, but his memory will remain in the hearts of the people who knew him.

Wieslaw A. Dudek

Below we present the list of publications of Valentin S. Trokhimenko.

1. W.A. Dudek, V.S. Trokhimenko, *Menger algebras of k-commutative n-place functions*, Georgian Math. J. (in print).
2. —— *Menger algebras of idempotent n-ary operations*, Studia Sci. Math. Hungar. 55 (2018), 260–269.
3. —— *Menger algebras of associative and self-distributive n-ary operations*, Quasigroups Related Systems, 26 (2018), 45–52.
4. —— *On σ-commutativity in Menger algebras of n-place functions*, Comm. Algebra 45 (2017), 4557–4568.
5. —— *On (i, j)-commutativity in Menger algebras of n-place functions*, Quasigroups Related Systems, 24 (2016), 219–230.
6. —— *On some subtraction Menger algebras of multiplace functions*, Semigroup Forum, 93 (2016), 375–386.
7. —— *De Morgan $(2, n)$-semigroups of n-place functions*, Comm. Algebra, 44 (2016), 4430–4437.

8. —— *The relations of semiadjacency and semicompatibility in n-semigroups of transformations*, Semigroup Forum 90 (2015), 113–125.

9. —— *Congruences on Menger algebras*, Comm. Algebra, 42 (2014), 3407–3426.

10. —— *Menger algebras of n-place interior operations*, Algebra Universalis, 70 (2013), 137–147.

11. —— *Algebras of multiplace functions*, De Gruyter, Berlin, 2012, 389 pp.

12. —— *Subtraction Menger algebras*, Semigroup Forum, 85 (2012), 111–128.

13. —— *Menger algebras of n-place functions*, Proc. Intern. Confer. on Algebra 2010, 198–218, World Sci. Publ., Hackensack, NJ, 2012.

14. V.S. Trokhimenko, *On n-groupoids in which all transformations are endomorphisms*, Quasigroups Related Systems, 19 (2011), 349–352.

15. W.A. Dudek, V.S. Trokhimenko, *The relation of semiadjacency on n-semigroups of transformations*, Semigroup Forum, 82 (2011), 109–120.

16. —— *Algebras of multiplace functions*, Christian Dawn, Kremenchuk, 2010, 286 pp.

17. —— *Stationary subsets of functional Menger n-algebras of multiplace functions*, Algebra Colloq. 17 (2010), 65–73.

18. —— *Stabilizers of functional Menger systems*, Comm. Algebra, 37 (2009), 985–1000.

19. —— *Projection representable relations on Menger* $(2, n)$*-semigroups*, Czechoslovak Math. J. 58(133) (2008), 1015–1037.

20. —— *Representations of* $(2, n)$*-semigroups by multiplace functions*, Studia Sci. Math. Hungar. 44 (2007), 131–146.

21. —— *Menger algebras of multiplace functions*, (Russian), Universitatea de Stat din Moldova, Chişinău, 2006, 238 pp.

22. —— *Representations of Menger* $(2, n)$*-semigroups by multiplace functions*, Comm. Algebra 34 (2006), 259–274.

23. —— *Functional Menger P-algebras*, Comm. Algebra 30 (2002), 5921–5931.

24. V.S. Trokhimenko, *Representations of positional algebras*, Quasigroups Related Systems, 8 (2001), 87–96.

25. —— *Algebras of vector-valued functions*, Quasigroups Related Systems, 8 (2001), 73–85.

26. —— *Stationary subsets and stabilizers of restrictive Menger P-algebras of multiplace functions*, Algebra Universalis, 44 (2000), 129–142.

27. —— *v-regular Menger algebras*, Algebra Universalis, 38 (1997), 150–164.

28. —— *On some Menger algebras of multiplace transformations of ordered sets*, Algebra Universalis, 33 (1995), 375–386.

29. —— *A characterization of iterative function algebras*, Algebra Universalis, 31 (1994), 542-550.

30. —— *Transformation semigroups of invertible transformations*, (Russian) Theory of semigroups and its applications, 8 (1987), 87–97, Saratov. Gos. Univ., Saratov.

31. —— *Characteristics of some algebras of functions of many-valued logic*, Cybernetics, 23 (1987), 362-368 (translation from Kibernetika (Kiev) 3 (1987), 63–67).

32. —— *Characterization of a semigroup of semiclosure operations on an ordered set*, (Russian) Ordered sets and lattices, No. 9, 100–105, Saratov. Gos. Univ., Saratov, 1986.

33. —— *Stabilizers of ordered Menger algebras*, Sov. Math. 30 (1986), No. 6, 112-114 (translation from Izv. Vyssh. Uchebn. Zaved. Mat. 6 (1986), 74–76).

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39. —— *Connectivity relation on ordered Menger algebras*, Ukr. Math. J. 35 (1093), 388-393 (translation from Ukrain. Mat. Zh. 35 (1983), no. 4, 461–466).

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46. —— *Semiadjacence relation in algebras of multiplace functions*, (Russian) Theory of semigroups and its applications, 3 (1974), 108–118, Izdat. Saratov. Univ., Saratov.

47. V.S. Trokhimenko, O.V. Kolesnikov, *Representations of Menger operatives*, (Russian) Study of approx. functions and their appl., 5 (1974), 190–199 (Dnipropetrovs State Univ.)

48. V.S. Trokhimenko, *Semiadjacence relation in algebras of multiplace functions*, (Russian) Theory of semigroups and its applications, 3 (1974), 108–118, Izdat. Saratov. Univ., Saratov.

49. —— *Positional systems of multiplace functions*, (Russian) Studies in the theory of quasigroups and loops, pp. 165–174, Izdat. Ştiinca, Kishinev, 1973.

50. —— *Menger's function systems*, (Russian) Izv. Vysš. Učebn. Zaved. Matematika, 11(138) (1973), 71–78.

51. —— *A characterization of Δ-algebras of multiplace functions*, (Russian) General systems theory, pp. 95–104. Akad. Nauk Ukrain. SSR Inst. Kibernet., Kiev, 1972.

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53. V.S. Trokhimenko, *The algebras of binary operations*, (Russian) Mat. Issled. 7 (1972), 253–261.

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