Quasigroups and Related Systems **29** (2021), 213 – 222

Semigroups in which 2-absorbing ideals are prime and maximal

Biswaranjan Khanra and Manasi Mandal

Abstract. We characterize commutative semigroups in which 2-absorbing ideals are maximal. We introduce the concept of 2-AB semigroups in which 2-absorbing ideals are prime and characterize 2-AB semigroups in terms of minimal prime ideal over a 2-absorbing ideal and study some properties of these semigroups.

1. Introduction

Throughout this paper all semigroups are commutative, prime ideals are proper and whenever speaking about maximal ideals we suppose, of course, it exists.

The notion of 2-absorbing ideals for commutative ring was introduced as a generalization of prime ideals by Badwai [1] and later extended to commutative semigroup by [5] and [3] as follows: A proper ideal I of a semigroup S is said to be a 2-*absorbing* ideal of S if for any elements $s_1, s_2, s_3 \in S$, $s_1s_2s_3 \in I$ implies $s_1s_2 \in I$ or $s_1s_3 \in I$ or $s_2s_3 \in I$. Clearly, every prime ideal is 2-absorbing but the converse is not true (see Lemma 2.1 and Example 2.2).

In this paper, we prove that every maximal ideal of a commutative semigroup is 2-absorbing but converse is not true (see Theorem 2.3). In [2], D. Bennis characterize commutative rings in which 2-absorbing ideals are prime. These observations prompted us to solve the following two natural questions:

(1) In which class of semigroups 2-absorbing ideals are maximal?

(2) In which class of semigroups 2-absorbing ideals are prime?

We establish an analogues result of Theorem 2.3 in a commutative ring (Theorem 2.4). Then we characterize the class of semigroups with unity (Theorem 2.7) and without unity (Theorem 2.11), in which 2-absorbing ideals are maximal. Next, we define the notion of 2-AB semigroup, in which 2-absorbing ideals are prime and an example of this semigroup is given (Definition 3.1 and Example 3.2). We study many properties of a 2-AB semigroup S such as 2-absorbing ideals are linearly ordered, S has atmost one maximal ideal, S is semiprimary and prime ideals of S are idempotent (Theorem 3.3). Then we characterize 2-AB semigroup in terms of minimal prime ideal over a 2-absorbing ideal (Theorem 3.5), some other characterizations have also been established (Theorem 3.6, Theorem 3.7 and

2010 Mathematics Subject Classification: 20M12, 20M14.
Keywords: Commutative semigroup, Prime ideal, Maximal ideal, 2-absorbing ideal.

Theorem 3.9). We study some equivalent conditions for a regular semigroup S to be 2-AB semigroup (Theorem 3.11). Finally, we prove that a semigroup S will be 2-AB if S is with unity and having no essential congrurence (Corollary 3.12) or every 2-absorbing ideal of S generated by idempotent (Theorem 3.13).

Before going to the main work we recall some preliminaries which are necessary:

A non-empty ideal P of a semigroup S is said to be *prime* if $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$, A, B being ideals of S. An ideal P is said to be *completely prime* if $ab \in P$ implies $a \in P$ or $b \in P$, a, b being elements of S. The concepts of prime and completely prime ideal are coincide if S is commutative.

For an ideal A of a semigroup S, a *radical* of A, denoted as $\sqrt{A}$, is the set of all $x \in S$ such that some power of x is in A. An ideal A of S is called *primary* if $ab \in A$ implies either $a \in A$ or $b \in \sqrt{A}$. An ideal I of a semigroup S is said to be *semiprimary* ideal if $\sqrt{I}$ is a prime ideal of S. A commutative semigroup S is called *fully prime semigroup* if every ideal of S is prime and *primary* if every ideal of S is primary. Also a semigroup S is said to be *semiprimary* if every ideal of S is a semiprimary ideal of S. A semigroup in which every ideal is idempotent is called a *fully idempotent semigroup*.

Theorem 1.1. (cf. [7]) *A commutative semigroup S is semiprimary if and only if prime ideals of S are linearly ordered.*

A commutative semigroup S is said to be *Archimedian* if, for any two elements of S, each divides some power of the other. In [10] it is proved that a commutative semigroup is archimedian if and only if S has no proper prime ideals.

We will use the following theorems proved in [11].

Theorem 1.2. *If I and J are any two ideals of a commutative semigroup S, then the following statements are true;*

(1) $IJ \subseteq I \cap J \subseteq I$.
(2) $I \subseteq \sqrt{I}$.
(3) $I \subseteq J \Rightarrow \sqrt{I} \subseteq \sqrt{J}$,
(4) $\sqrt{IJ} = \sqrt{(I \cap J)} = \sqrt{I} \cap \sqrt{J}$,
(5) *If A is a prime ideal of S, then $\sqrt{A} = A$ and if A is a primary ideal of S, then $\sqrt{A}$ is a prime ideal of S.*

Theorem 1.3. *Let A be an ideal of a commutative semigroup S with unity. If $\sqrt{A} = M$, where M is a maximal ideal of S, then A is a primary ideal of S.*

Theorem 1.4. *In a commutative semigroup S with unity, the unique maximal ideal M is prime, which is the union of all proper ideals of S; $\sqrt{M^n} = M$ for every positive integer n and M^n is a primary ideal for every positive integer n.*

Theorem 1.5. *The radical of an ideal I in a commutative semigroup is the intersection of all prime ideals containing I.*

Theorem 1.6. *Any prime ideal containing an ideal I in a semigroup contains a minimal prime ideal belonging to I.*

Also the following theorem will be used.

Theorem 1.7. (cf. [12]) *If M is a maximal ideal of a semigroup S such that the complement of M contains either more than one element, or an idempotent, then M is a prime ideal of S.*

2. The case when 2-absorbing ideals are maximal

Lemma 2.1. *In a commutative semigroup every prime ideal is 2-absorbing.*

Proof. Let I be a prime ideal of S and $abc \in I$ with $ab \notin I$ for some $a, b, c \in S$. Since I is prime, so $c \in I$, which implies $ac \in I$ and $bc \in I$. So I is a 2-absorbing ideal of S. □

The following example shows that the converse of the above lemma is not true:

Example 2.2. *The principal ideal $I = (6)$ in the semigroup $S = (\mathbb{N}, \cdot)$ is 2-absorbing but not prime as $2 \cdot 3 \in (6)$ but neither $2 \in (6)$ nor $3 \in (6)$.*

A commutative semigroup with unity has a unique maximal ideal, which is prime and 2-absorbing. But in a commutative semigroup without unity maximal ideal need not be prime. For example, the ideal $I = \{m \in \mathbb{N} : m \geqslant 2\}$ in the semigroup $S = (\mathbb{N}, +)$ is maximal but not prime.

Theorem 2.3. *In a commutative semigroup without unity every maximal ideal is 2-absorbing.*

Proof. Let M be a maximal ideal of a semigroup S without unity and $abc \in M$ with $ab \notin M$ for some $a, b, c \in S$.

1. If $c \in M$ then $ac \in M$ and $bc \in M$, since M is an ideal of S. Hence M is a 2-absorbing ideal of S.

2. Let $c \notin M$. Since $ab \notin M$, then both a, b belongs to $S - M$. Now if $c \neq ab$, then $S - M$ contains two distinct elements c and ab. Again if $c = ab$ and $a \neq b$ then $S - M$ contains two distinct elements a and b and if $a = b$ then $\{a, a^2\}$ belongs to $S - M$, moreover if $a = a^2$, then a is an idempotent element of S. Thus in either case $S - M$ contains more than one elemenet or an idempotent, hence M is a prime ideal of S by Theorem 1.7. Consequently, M is a 2-absorbing ideal of S by Lemma 2.1. □

The converse is not true if S has unity. Indeed, the ideal $I = \{m \in S : m \geqslant 2\}$ in $S = (\mathbb{N} \cup \{0\}, +)$ is 2-absorbing but not maximal.

Theorem 2.4. *In a commutative ring every maximal ideal is 2-absorbing.*

Proof. Let M be a maximal ideal of a commutative ring R and $abc \in M$ with $ab \notin M$, for some $a, b, c \in R$. If $c \notin M$, then $M + (c) = R = M + (ab)$, where (c) and (ab) denotes respectively the principal ideal generated by c and ab.

Since $a, b \in R$, so there exist $r, s \in R$ and $p, q \in \mathbb{Z}$ such that $a = m+rc+pc$ and $b = n+sab+qab$, for some $m, n \in M$. Therefore $ab = (m+rc+pc)(n+sab+qab) = mn+msab+qmab+nrc+rsabc+qrabc+pnc+psabc+pqabc \in M$, a contradiction. Hence $c \in M$ implies $ac, bc \in M$ and consequently M is 2-absorbing. □

The converse is not true. In the commutative ring $\mathbb{Z}[x]$ with unity the principal ideal (x) is 2-absorbing but it is not maximal.

Lemma 2.5. *The intersection of any two prime ideals is a* 2*-absorbing ideal.*

Proof. Let $abc \in P_1 \cap P_2$ for some $a, b, c \in S$. Then $abc \in P_1$ and $abc \in P_2$. Since P_1 and P_2 are prime ideals so either $a \in P_1$ or $b \in P_1$ or $c \in P_1$ and also either $a \in P_2$ or $b \in P_2$ or $c \in P_2$. Thus in either ab or bc or ac belongs to $P_1 \cap P_2$. □

Theorem 2.6. *If in a semigroup S all* 2*-absorbing ideals are maximal, then S has at most one prime ideal. This ideal is maximal.*

Proof. By Lemma 2.5 the intersection of two prime ideals P_1 and P_2 is a 2-absorbing ideal. It is maximal and it is contained in both ideal P_1 and P_2. Hence $P_1 = P_2$. □

Theorem 2.7. *In a semigroup S with unity every* 2*-absorbing ideal is maximal if and only if S is either a group or S has a unique* 2*-absorbing ideal A such that $S = A \cup H$, where H is the group of units and A is an archimedian subsemigroup of S.*

Proof. Let S be a semigroup with unity in which every 2-absorbing ideal is maximal. If S is not group, then S has a unique maximal ideal A which is the only prime as well as 2-absorbing ideal of S. Therefore $S = A \cup H$, where A is unique 2-absorbing ideal of S and H is the group of units. Since A is the unique prime ideal in S, for any $p,q \in A$, $\sqrt{(p)} = \sqrt{(q)} = A$. Then there exist positive integers m and n such that $p^m = qx$ and $q^n = py$ for some x, $y \in S$. So $p^{m+1} = q(px)$ and $q^{n+1} = p(qy)$, where $px, qy \in M$. Hence A is an archimedian subsemigroup of S.

Conversely, let A be the unique 2-absorbing ideal of S. Since in a semigroup with unity has unique maximal ideal and maximal ideals are 2-absorbing, therefore A is maximal, as desired. □

Theorem 2.8. *Let S be a regular semigroup with unity such that every* 2*-absorbing ideal is of the form M^n, where n is any positive integer and M is the unique maximal ideal of S. Then an ideal I of S is a primary if and only if I is a* 2*-absorbing ideal of S.*

Proof. Let I be a 2-absorbing ideal of a semigroup S with unity, which is of the form M^n, where n is any positive integer and M is the unique maximal ideal of S. Then $\sqrt{I} = \sqrt{M^n} = M$ by Theorem 1.4. Hence I is a primary ideals of S.

Conversely, let I be a primary ideal of S. Since S is regular so $I = \sqrt{I}$. Cosequently I is prime and hence I is 2-absorbing ideal of S. □

As a consequence of the above theorem and Theorem 2.1 of [9] we obtain

Corollary 2.9. *If in a regular semigroup S with zero and identity every 2-absorbing ideal has the form M^n, where $n \in \mathbb{N}$ and M is the maximal ideal of S, then every non-zero 2-absorbing ideal of S is maximal if and only if*

(i) *S is the union of two groups with adjoined zero, or*

(ii) *$S = H \cup M$, where $M = \{0, xh : h \in H, x^2 = 0, x \in M\}$ and H is the group of units.*

Theorem 2.10. *If in a semigroup S with unity all 2-absorbing ideals are maximal, then*

(1) *S is a primary semigroup,*

(2) *$M^2 = M$, where M is the maximal ideal of S,*

(3) *S has atmost one idempotent different from identity.*

Proof. (1). Let S be a semigroup with unity in which all 2-absorbing ideals are maximal. Then S has a unique maximal ideal, say M, which is the union of all proper ideals of S and it is also the unique prime ideal of S. Then for any ideal I of S, $\sqrt{I} = M$, hence I is a primary ideal of S. Therefore S is a primary semigroup.

(2). Let $abc \in M^2 \subseteq M$ for some $a, b, c \in S$. Since M is a prime ideal of S either a or b or c belongs to M. Let $a \in M$. Then $bc \in M$, implies $b \in M$ or $c \in M$. Hence ac or ab belongs to M^2 and so M^2 is a 2-absorbing ideal of S. Since every 2-absorbing ideal of S is maximal so M^2 is a maximal ideal of S. Therefore $M^2 = M$.

(3). If e and f are idempotents different from the identity, then $\sqrt{(eS)} = \sqrt{(fS)} = M$, where M is the unique prime as well as unique maximal ideal of S. Therefore $e = ef = f$. □

Theorem 2.11. *Let S be a semigroup without unity. Then 2-absorbing ideals of S are maximal if and only if complement of each 2-absorbing ideals contains exactly one non-idempotent element or is a subgroup of S.*

Proof. Let S be a semigroup without unity in which 2-absorbing ideals are maximal. Then S has at most one prime ideal (Theorem 2.6). Let I be a 2-absorbing ideal of S but not prime. Now if complement of I in S contains more than one element or an idempotent, then I is prime (Theorem 1.7), a contradiction. Hence in this case complement of a 2-absorbing ideal contains exactly one non-idempotent element of S. Again, let a 2-absorbing ideal J is prime. Then $a, b \in S - I$ implies $ab \in S - I$, since I is a prime ideal of S. We know that complement of a maximal ideal in a commutative semigroup is a $\mathcal{H}$-class (Green's), and a, b, ab all belong to same $\mathcal{H}$-class $S - I$ of the semigroup S. Hence $S - I$ is a subgroup of S (Theorem 2.16, [4]), as desired.

Conversely, if complement of a 2-absorbing ideal contains exactly one element then clearly it is maximal. Now let complement of a 2-absorbing ideal J forms a subgroup of S. If J is not maximal, then J is contained in a proper ideal K of S.

Let i be the identity element of $S - J$. Since $J \neq K$, there exists $p \in K - J$ such that $pq = i$ for some $q \in S$. Hence $i \in K$. Since $K \neq S$, there exists $m \in S - K$ such that $m = mi \in K$, a contradiction. Thus, J is a maximal ideal of S. □

Since in an archimedian semigroup has no prime ideal, we have

Corollary 2.12. *In an archimedian semigroup S without unity all 2-absorbing ideals are maximal if and only if complement of every 2-absorbing ideal contains exactly one non-idempotent element.*

Corollary 2.13. *In a semigroup S without unity all 2-absorbing ideals are prime as well as maximal if and only if the complement of each 2-absorbing ideal is a subgroup of S.*

3. The case when 2-absorbing ideals are prime

In this section we characterize the class of semigroups in which 2-absorbing ideals are prime and study some properties of this semigroup.

Definition 3.1. A commutative semigroup S is said to be a 2-*AB semigroup* if every 2-absorbing ideal of S is prime.

Example 3.2. In a semigroup $S = \{a, b\}$ with the multiplication determined by $a^2 = a$, $b^2 = b$, $ab = ba = a$, $\{a\}$ is a 2-absorbing ideal which also is prime. Hence S is a 2-AB semigroup.

Theorem 3.3. *Let S be a 2-AB semigroup. Then*

(1) *2-absorbing ideals of S are linearly ordered,*

(2) *prime ideals of S are linearly ordered,*

(3) *S has at most one maximal ideal, if exists then it is prime,*

(4) *S is a semiprimary semigroup,*

(5) *idempotents in S form a chain under natural ordering,*

(6) *$P = P^2$ for every prime ideal P of S,*

(7) *semiprime ideals of S are prime.*

Proof. (1). Let A and B be any two distinct 2-absorbing ideals of a 2-AB semigroup S. So $A \cap B$ is 2-absorbing (Lemma 2.5) and hence prime, which implies either $A \subseteq B$ or $B \subseteq A$.

(2) Clearly prime ideals of S are linearly ordered.

(3) Let M_1 and M_2 be two maximal ideal of S. Since every maximal ideal of S is 2-absobing (Theorem 2.3), so $M_1 \subseteq M_2$ or $M_2 \subseteq M_1$ which implies $M_1 = M_2$. Hence S has atmost one maximal ideal and if exists clearly it is prime.

(4) By Theorem 1.1, a commutative semigroup is semiprimary if and only if prime ideals are linearly ordered. Hence S is a semiprimary semigroup.

(5) Since S is a semiprimary semigroup, then for any ideal A of S, $\sqrt{A}$ is prime. Let e and f are any two idempotents of S. Then $\sqrt{eS}$ and $\sqrt{fS}$ are prime ideals, so either $\sqrt{eS} \subseteq \sqrt{fS}$ or $\sqrt{fS} \subseteq \sqrt{eS}$, which proves that the idempotents form a chain under natural ordering.

(6) Let P be a prime ideal of S and $abc \in P^2 \subseteq P$ for some $a, b, c \in S$. Since P is a prime ideal of S, either $a \in P$ or $b \in P$ or $c \in P$. Let $a \in P$. Then $bc \in P$, implies b or c belogs to P and so ac or ab belongs to P^2. Hence P^2 is a 2-absorbing ideal of S and so P^2 is a prime ideal of S. Let $x \in P$. Then $x^2 \in P^2$ implies $x \in P^2$ so $P \subseteq P^2$. Therefore $P = P^2$.

(7) Let I be a semiprime ideal of S. Then $I = \sqrt{I}$ is a prime ideal of S, since prime ideals of S are linearly ordered, as desired. □

Lemma 3.4. *Let S be a semigroup with unity and unique maximal ideal M. Then for every prime ideal P, PM is a 2-absorbing ideal of S. Moreover, PM is prime if and only if $PM = P$.*

Proof. Let $xyz \in PM \subseteq P$. Since P is prime, either $x \in P$ or $y \in P$ or $z \in P$. Let $x \in P$. Then either $y \in M$ or $z \in M$, since M is also prime. Hence $xy \in PM$ or $xz \in PM$. Consequently, PM is a 2-absorbing ideal of S. Clearly, PM is prime if and only if $PM = P$. □

The following is a characterization of a 2-AB semigroup in terms of minimal prime ideal over a 2-absorbing ideal, which is analogous to (Theorem 2.3, [2]).

Theorem 3.5. *A semigroup S with unity is a 2-AB semigroup if and only if prime ideals of S are linearly ordered and if P is a minimal prime ideal over a 2-absorbing ideal I, then $IM = P$, where M is the unique maximal ideal of S.*

Proof. Let I be a 2-absorbing ideal of a 2-AB semigroup S with unity. Then prime ideals of S are linearly ordered (Theorem 3.3) and I is prime by hypothesis. Then $IM = I$ (Lemma 3.4).

Conversely, let I be a 2-absorbing ideal of S. Since prime ideals are linearly ordered and $P = IM$, where P is a minimal prime ideal over I, $P = IM \subseteq I \cap M = I \subseteq P$ implies $I = P$, as desired. □

Theorem 3.6. *A commutative semigroup S is a 2-AB semigroup if and only if $P = P^2$ for every prime ideal P of S and every 2-absorbing ideal of S is of the form A^2, where A is a prime ideal of S.*

Proof. Let P be a 2-absorbing ideal of a 2-AB semigroup. Then P is prime and so $P = P^2$ (Theorem 3.3(6)).

Conversely, let I be a 2-absorbing ideal of S. Then $I = A^2 = A$, where A is a prime ideal of S. □

Theorem 3.7. *A commutative semigroup S is a 2-AB semigroup if and only if its prime ideals are linearly ordered and $A = A^2$ for every 2-absorbing ideal A of S.*

Proof. Let S be a 2-AB semigroup. Let P_1 and P_2 be two prime ideals of S. Then $P_1 \cap P_2$ is 2-absorbing ideal of S (Lemma 2.5) and so prime, which implies either $P_1 \subseteq P_2$ or $P_2 \subseteq P_1$. Again let A be a 2-absorbing ideal of S and so prime. Therefore $A = A^2$ (Theorem 3.3).

Conversely, let A be any 2-absorbing ideal of S and $x \in \sqrt{A}$. Then $x^2 \in A = A^2$, since A is 2-absorbing ideal of S. This implies $x \in A$, so $A = \sqrt{A}$. Since prime ideals are linearly ordered so A is prime and hence S is a 2-AB semigroup. □

Since in a fully idempotent semigroup S, $A = A^2$ for every ideal A of S, the following is a simple consequence of above theorems:

Corollary 3.8. *A fully idempotent semigroup S is a 2-AB semigroup if and only if one of the following conditions hols:*

(1) *Prime ideals are linearly ordered.*

(2) *Every 2-absorbing ideal is of the form P^2, where P is a prime ideal of S.*

Theorem 3.9. *A semigroup S is a 2-AB semigroup if and only if its prime ideals are linearly ordered and $A = \sqrt{A}$ for every 2-absorbing ideal A of S.*

Proof. Let S be a 2-AB semigroup. Then prime ideals of S are linearly ordered (Theorem 3.3). Again any 2-absorbing ideal A of S is prime so $A = \sqrt{A}$.

Conversely, let A be a 2-absorbing ideal of S. Then $A = \sqrt{A} = \bigcap P_i = P_\beta$, for some $\beta \in \Lambda$ and where $\{P_i : i \in \Lambda\}$ are prime ideals containing A. Hence S is a 2-AB semigroup. □

Since in a semiprimary semigroup prime ideals are linearly ordered (Theorem 1.1), the following corollary is an obvious consequence of the above theorem:

Corollary 3.10. *A semiprimary semigroup S is a 2-AB semigroup if and only if $A = \sqrt{A}$ for every 2-absorbing ideal A of S.*

Theorem 3.11. *For a commutative regular semigroup S the following statements are equivalent:*

(1) *S is 2-AB semigroup.*

(2) *2-absorbing (prime) ideals are linearly ordered.*

(3) *Idempotents in S form a chain under natural ordering.*

(4) *All ideals of S are linearly ordered.*

(5) *S is a fully prime semigroup.*

(6) *S is a primary semigroup.*

(7) *S is a semiprimary semigroup.*

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3) by Theorem 3.3.

(3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (7) follows from Theorem 2.4 of [11].

(7) $\Rightarrow$ (1). Let A be a 2-absorbing ideal of a commutative regular semigroup S. Then $A = \sqrt{A} = \bigcap P_\alpha$, where $\{P_\alpha : \alpha \in \Lambda\}$ are the prime ideals of S contaning

A. Since S is semiprimary, so prime ideals are linearly ordered, which implies $A = \sqrt{A} = P_\beta$ for some $\beta \in \Lambda$. Therefore S is a 2-AB semigroup. □

Let $\mathcal{D}$ be the class of commutative semigroups with an identity element and having no proper essential congruences, i.e. congruences δ such that $\alpha \cap \delta \neq i$ for every congruence $\alpha \neq i$, where i is the identity relation on S. Oehmke [8], proved that if $S \in \mathcal{D}$, then the set of ideals of S are linearly ordered by inclusion and hence the set of prime ideals of S are linearly ordered. Again Khaksari [6], proved that if $S \in \mathcal{D}$, then S is regular i.e. $A = \sqrt{A}$ for every ideal A of S. So as a simple consequence of Theorem 3.9, we have the following result:

Corollary 3.12. *If $S \in \mathcal{D}$, then S is a 2-AB semigroup.*

Theorem 3.13. *If every 2-absorbing ideal of a semigroup S has an idempotent generator, then S is a 2-AB semigroup.*

Proof. Let I be a 2-absorbing ideal of S generated by the idempotent e i.e. $I = (e) = eS$. Since S is commutative so $I = I^2$. It is clear that $I \subseteq \sqrt{I}$. Let $x \in \sqrt{I}$. Then $x^2 \in I = I^2$, since I is 2-absorbing. This implies $x \in I$, so $\sqrt{I} \subseteq I$. Hence $I = \sqrt{I}$. Again, let P, Q be two prime ideals of S. Then the prime ideal $P \cup Q$ is 2-absorbing, has an idempotent generator e, i.e. $P \cup Q = eS$. But then $e \in P$ or $e \in Q$. This implies either $P = eS$ or $Q = eS$ and either $P \subseteq Q$ or $Q \subseteq P$. Hence by Theorem 3.9, S is a 2-AB semigroup. □

Since every principal ideal of a commutative regular semigroup has an idempotent generator, the following is an obvious consequence of the above theorem:

Corollary 3.14. *If every 2-absorboing ideal of a commutative regular semigroup S is principal, then S is a 2-AB semigroup.*

Acknowledgements. The authors are grateful to prof. Sujit Kumar Sardar of Jadavpur University for constant encouragement and active guidence throughout the preparation of the paper. We are also thankful to the anonymous referee for making some valuable suggesitions.

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Received December 10, 2020

Department of Mathematics, Jadavpur University, Kolkata-700032, India
Emails: biswaranjanmath91@gmail.com (B. Khanra), manasi_ju@yahoo.in (M. Mandal)