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A new characterization of orthogonal simple groups $\mathbf{B_2(2^{4n})}$

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Abstract. We prove that orthogonal simple groups $B_2(q)$, where $q = 2^{4n}$ and $q^2 + 1$ is a prime number can be uniquely determined by the order of the group and the number of elements with the same order.

1. Introduction

Let G be a finite group, $\pi(G)$ be the set of prime divisors of order of G and $\pi_e(G)$ be the set of elements order in G. If $k \in \pi_e(G)$, then we denote the set of the number of elements of order k in G by $m_k(G)$ and the set of the number of elements with the same order in G by $nse(G)$. In other words, $nse(G) = \{m_k(G) | k \in \pi_e(G)\}$. Also, we denote a Sylow p-subgroup of G by G_p and the number of Sylow p-subgroups of G by $n_p(G)$. Throughout this paper, we denote by ϕ the Euler's totient function. The prime graph $\Gamma(G)$ of group G is a graph whose vertex set is $\pi(G)$, and two distinct vertices u and v are adjacent if and only if $uv \in \pi_e(G)$. Moreover, assume that $\Gamma(G)$ has $t(G)$ connected components π_i, for $i = 1, 2, \dots, t(G)$. In the case where G is of even order,we assume that $2 \in \pi_1$.

Characterization of groups by $nse(G)$ is one of problems that related to Thompson's problem (see[17, Problem 12.37]). Next, in the way the authors in ([6, 7, 8, 4, 5, 13, 12, 11, 14, 18, 20], proved that some of groups are characterizable by the order of groups and the number of elements with the same order. The groups, such as sporadic groups, S_p, where p is a prime, suzuki groups, simple K_4-groups, ${}^2G_2(q)$, where $q \pm \sqrt{3q} + 1$ are prime numbers, $L_2(p)$, where p is a prime and $L_2(2^n)$ where $2^n - 1$ or $2^n + 1$ is prime number, the symplectic group $C_2(3^n)$, where $n = 2k$ $(k \geqslant 0)$ and $(\frac{3^{2n}+1}{2})$ is

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a prime number and $L_3(q)$ where $0 < q = 5k \pm 2$, $(k \in \mathbb{Z})$ and $q^2 + q + 1$ is a prime number. In this paper, we prove that orthogonal simple groups $B_2(q)$, where $q = 2^{4n}$ and $q^2 + 1$ is a prime number can be uniquely determined by the order of group and the number of elements with the same order. In fact, we prove the following main theorem. .

Main Theorem. *Let G be a group with $|G| = |B_2(q)|$ and $nse(G) = nse(B_2(q))$, where $q = 2^{4n}$ and $p = q^2 + 1$ is a prime number. Then $G \cong B_2(q)$.*

2. Notation and Preliminaries

Lemma 2.1. [10, Theorem10.3.1] *Let G be a Frobenius group with kernel K and complement H. Then*

1, *$t(G) = 2$, $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$;*

2. *$|H|$ divides $|K| - 1$;*

3. *K is nilpotent.*

Definition 2.2. A group G is called a 2-*Frobenius group* if there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/H and K are Frobenius groups with kernels K/H and H respectively.

Lemma 2.3. [1, Theorem 2] *Let G be a 2-Frobenius group of even order. Then*

1. *$t(G) = 2$, $\pi(H) \cup \pi(G/K) = \pi_1$ and $\pi(K/H) = \pi_2$;*

2. *G/K and K/H are cyclic groups satisfying $|G/K|$ divides $|Aut(K/H)|$.*

Lemma 2.4. [25, Theorem A] *Let G be a finite group with $t(G) \geqslant 2$. Then one of the following statements holds:*

1. *G is a Frobenius group;*

2. *G is a 2-Frobenius group;*

3. *G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group, H is a nilpotent group and $|G/K|$ divides $|Out(K/H)|$.*

Lemma 2.5. [9] *Let G be a finite group and m be a positive integer dividing $|G|$. If $L_m(G) = PSL_m(G) = \{g \in G \mid g^m = 1\}$, then $m \mid |L_m(G)|$.*

Lemma 2.6. *Let G be a finite group. Then for every $i \in \pi_e(G)$, $\varphi(i)$ divides $m_i(G)$, and i divides $\sum_{j|i} m_j(G)$. Moreover, if $i > 2$, then $m_i(G)$ is even.*

Proof. By Lemma 2.5, the proof is straightforward. $\square$

Lemma 2.7. [26, Lemma 6(iii)] *Let q, k, l be natural numbers. Then*

1. $(q^k - 1, q^l - 1) = q^{(k,l)} - 1$.
2. $(q^k + 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1 & \text{if both } \frac{k}{(k,l)} \text{ and } \frac{l}{(k,l)} \text{ are odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$
3. $(q^k - 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1 & \text{if } \frac{k}{(k,l)} \text{ is even and } \frac{l}{(k,l)} \text{ is odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$

In particular, for every $q \geqslant 2$ and $k \geqslant 1$, the inequality $(q^k - 1, q^k + 1) \leqslant 2$ holds.

Lemma 2.8. *Let G be a simple groups $B_2(q)$, where $q = 2^{4n}$ and $p = q^2 + 1$ is a prime number. Then $m_p(G) = (p-1)|G|/(4p)$ and for every $i \in \pi_e(G) - \{1, p\}$, p divides $m_i(G)$.*

Proof. Since $|G_p| = p$, we deduce that G_p is a cyclic group of order p. Thus $m_p(G) = \varphi(p)n_p(G) = (p-1)n_p(G)$. Now it is enough to show that $n_p(G) = |G|/(4p)$. By [16], p is an isolated vertex of $\Gamma(G)$. Hence $|C_G(G_p)| = p$ and $|N_G(G_p)| = \alpha p$ for a natural number α. We know that $N_G(G_p)/C_G(G_p)$ embeds in $Aut(G_p)$, which implies $\alpha \mid p - 1$. Furthermore, by Sylow's Theorem, $n_p(G) = |G : N_G(G_p)|$ and $n_p(G) \equiv 1 \pmod{p}$. Therefore p divides $|G|/(\alpha p) - 1$. Thus $q^2 + 1$ divides $q^4(q^4 - 1)(q^2 - 1)/(\alpha p) - 1$. It follows that $q^2 + 1$ divides $(q^8 - 2q^6 + q^4 - \alpha)$, hence $q^2 + 1$ divides $(q^2 + 1)(q^6 - 3q^4 + 4q^2 - 4) + (4 - \alpha)$, and since $\alpha \mid p - 1$, we obtain that $\alpha = 4$. Let $i \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, we conclude that $p \nmid i$ and $pi \notin \pi_e(G)$. Thus G_p acts fixed point freely on the set of elements of order i by conjugation and hence $|G_p| \mid m_i(G)$. So we conclude that $p \mid m_i(G)$. $\square$

3. Proof of the Main Theorem

In this section, we prove the main theorem by the following lemmas. We denote the simple groups $B_2(2^{4n})$, where $(2^{4n}+1)$ is a prime number by B and prime number $(2^{4n}+1)$ by p. Recall that G is a group with $|G| = |B|$ and $nse(G) = nse(B)$.

Lemma 3.1. *$m_2(G) = m_2(B)$, $m_p(G) = m_p(B)$, $n_p(G) = n_p(B)$, p is an isolated vertex of $\Gamma(G)$ and $p \mid m_k(G)$ for every $k \in \pi_e(G) - \{1, p\}$.*

Proof. By Lemma 2.6, for every $1 \neq r \in \pi_e(G)$, $r = 2$ if and only if $m_r(G)$ is odd. Thus we deduce that $m_2(G) = m_2(B)$. According to Lemma 2.6, $(m_p(G), p) = 1$. Thus $p \nmid m_p(G)$ and also Lemma 2.8 implies that $m_p(G) \in \{m_1(B), m_2(B), m_p(B)\}$. Moreover, $m_p(G)$ is even, so we conclude that $m_p(G) = m_p(B)$. Since G_p and B_p are cyclic groups of order p and $m_p(G) = m_p(B)$, we deduce that $m_p(G) = \varphi(p)n_p(G) = \varphi(p)n_p(B) = m_p(B)$, so $n_p(G) = n_p(B)$.

Now we prove that p is an isolated vertex of Γ(G). Assume the contrary. Then there is $t \in \pi(G) - \{p\}$ such that $tp \in \pi_e(G)$. So $m_{tp}(G) = \varphi(tp)n_p(G)k$, where k is the number of cyclic subgroups of order t in $C_G(G_p)$ and since $n_p(G) = n_p(B)$, it follows that $m_{tp}(G) = (t-1)(p-1)|B|k/(4p)$. If $m_{tp}(G) = m_p(B)$, then $t = 2$ and $k = 1$. Furthermore, Lemma 2.5 yields $p \mid m_2(G) + m_{2p}(G)$ and since $m_2(G) = m_2(C)$ and $p \mid m_2(C)$, we have $p \mid m_{2p}(G)$, which is a contradiction. So Lemma 2.8 implies that $p \mid m_{tp}(G)$. Hence $p \mid t-1$ and since $m_{tp}(G) < |G|$, we deduce that $t-1 < 5$. In conclusion we deduce that $t \in \{3, 4, 5\}$. Now since $p \nmid m_{tp}(G)$, this is a contradiction.

Let $k \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, $p \nmid k$ and $pk \notin \pi_e(G)$. Thus G_p acts fixed point freely on the set of elements of order k by conjugation and hence $|G_p| \mid m_k(G)$. So, we conclude that $p \mid m_k(G)$. □

Lemma 3.2. *The group G is neither a Frobenius group nor a 2-Frobenius group.*

Proof. Let G be a Frobenius group with kernel K and complement H. Then by Lemma 2.1, $t(G) = 2$ and $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$ and $|H|$ divides $|K| - 1$. Now by Lemma 3.1, p is an isolated vertex of $\Gamma(G)$. Thus we deduce that (i) $|H| = p$ and $|K| = |G|/p$, or (ii) $|H| = |G|/p$ and $|K| = p$. Since $|H|$ divides $|K|-1$, we conclude that the last case can not occur. So $|H| = p$ and $|K| = |G|/p$, hence $(q^2+1) \mid \frac{q^4(q^4-1)}{(q^2+1)} - 1$. So we conclude that $(q^2+1) \mid ((q^2+1)(q^6-3q^4+4q^2-4)+3$. Thus, $q^2+1 \mid 3$, which is impossible.

We now show that G is not a 2-Frobenius group. Let G be a 2-Frobenius group. Then, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/H and K are Frobenius groups by kernels K/H and H, respectively. Set $|G/K| = x$. Since p is an isolated vertex of $\Gamma(G)$, we have $|K/H| = p$ and $|H| = |G|/(xp)$. By Lemma 2.3, $|G/K|$ divides $|Aut(K/H)|$. Thus $x \mid p-1$ and since, by Lemma 2.7, $(p-1, q-1) = 1$, we have $(q^2, q-1) = 1$. Now since $|G/K||(p-1)$, we deduce that $q-1 \mid |H$. The group H is nilpotent. Therefore $H_t \rtimes K/H$ is a Frobenius group with kernel H_t and complement K/H, where $t = q-1$. So $|K/H|$ divides $|H_t|-1$. It implies that $q^2+1 \leqslant q-2$, but this is a contradiction. □

Lemma 3.3. *The group G is isomorphic to the group B.*

Proof. By Lemma 3.1, p is an isolated vertex of $\Gamma(G)$. Thus $t(G) > 1$ and G satisfies one of the cases of Lemma 2.4. Now Lemma 3.2 implies that G is neither a Frobenius group nor a 2-Frobenius group. Thus only the case (c) of Lemma 2.4 occurs. So G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group H is nilpotent, and G/K divides $|Out(K/H)|$. Note that since $p \mid |K/H|$ and p is an isolated vertex of $\Gamma(K/H)$, it follows that K/H is a simple C_{pp}-group with $p = 2^{8n}+1$. Now by [3] we have that K/H must be one of the following

(i) $Alt(p')$, $Alt(p'+1)$, $Alt(p'+2)$,
(ii) $A_1(r)$, $r = 2^{8n}$; p^k; $2.p^k \pm 1$ which is a prime, $8n \geqslant k = 1$,
(iii) $F_4(2^{2n})$,
(iv) ${}^2D_{8n+1}(2)$,
(v) ${}^2D_{a/2}(2^b)$; $ab = 16n$,
(vi) $B_a(2^b)$, $ab = 8n$; $a \geqslant 2$.

We go through all these cases.

Case (i). Suppose K/H is isomorphic to $Alt(p')$; $Alt(p'+1)$, or $Alt(p'+2)$. Note that $p \mid |K/H| \mid |G|$, so we consider $p = p'$ it follows that $2^{8n}+1 = p'$ then $2^{8n}+2 = p'+1$, but $2^{8n}+2 \nmid |A_n| \mid |G|$, we have a contradiction. Now if $p = p'-2$, then $2^{8n}+1 = p'-2$. As a result $2^{8n}+3 = p'$, again we have $2^{8n}+3 \nmid |A_n| \mid |G|$, so we have a contradiction.

Case (ii). Suppose that K/H is isomorphic to $A_1(r)$ with $r = 2^{8n'}$; p; $2p \pm 1$, for these last two possibilities r must be a prime. First, note that

$|A_1(2^{8n'})| = 2^{8n'}(2^{16n'}-1)$. On the other hand we have $p = r \pm 1$. So $2^{8n}+1 = 2^{8n'} \pm 1$. First if $2^{8n}+1 = 2^{8n'}+1$, so $n = n'$. Now, we know that $|A_1(r)| \mid |G|$, so $2^{8n}(2^{16n}-1) \mid 2^{16n}(2^{16n}-1)(2^{8n}-1)$. On the other hand, we have $2^{8n} = |K/H|_r \leq |G|_r \leq 2^{3n}$, which is a contradiction. For, other cases we have a contradiction.

Case (iii). Suppose K/H is isomorphic to $F_4(q')$, where $q' = 2^{2n'}$. So, we consider $p = q'^4+1$, $q'^4-q'^2+1$. Now if $p = q'^4+1$, then have $2^{8n}+1 = q'^4+1$. Thus $2^{8n} = 2^{8n'}$, as a result $n = n'$. On the otherhand, we know $|F_4(q')| \mid |G|$, so $q'^{24}(q'^{12}-1)(q'^8-1)(q'^6-1)(q'^2-1) \mid q^4(q^4-1)(q^2+1)$. It follows that $2^{48n'}(2^{24n'}-1)(2^{16n'}-1)(2^{12n'}-1)(2^{4n'}-1) \mid 2^{16n'}(2^{16n'}-1)(2^{8n'}+1)$, which is a contradiction. Now, we consider $p = q'^4-q'^2+1$, so $2^{8n}+1 = q'^4-q'^2+1$. It follows that $2(2^{8n-1}) = q'^2(q'^2-1)$. Thus, $q'^2-1 = 2$ and $q'^2 = 2^{8n-1}$. As a result $2^{4n'} = 3$ and $2^{4n'} = 2^{8n-1}$, which is a contradiction.

Case (iv). Suppose that K/H is isomorphic (iv) ${}^2D_{8n'+1}(2)$, so we consider $p = 2^{8n'+1}+1$. It follows that $2^{8n}+1 = 2^{8n'+1}+1$. Thus, we deduce $8n = 8n'+1$, but $|{}^2D_{8n}(2)| \nmid |G|$, which is a contradiction.

Case (v). Suppose that K/H is isomorphic ${}^2D_{a/2}(2^b)$; $ab = 16n$. Now, we consider $a = 4, 8, 16$ so we have the following possibilities.

(1). Let $K/H \cong {}^2D_2(2^{4n'})$. So, we consider $p = 2^{8n}+1 = 2^{8n'}+1$. It follows that $n = n'$. On the other hand, $|{}^2D_2(2^{4n'})| \mid |G|$, so $2^{8n'}(2^{8n'}-1)^2 \mid 2^{16n}(2^{16n}-1)(2^{8n}+1)$. It follows that $2^{8n}(2^{8n}-1)^2 \mid 2^{16n}(2^{16n}-1)(2^{8n}+1)$, which is a contradiction.

(2). Let $K/H \cong {}^2D_4(2^{2n'})$. So, we consider $p = 2^{8n}+1 = 2^{8n'}+1$. It follows that $n = n'$. On the other hand,$|{}^2D_4(2^{2n'})| \mid |G|$, so $2^{24n'}(2^{8n'}-1)(2^{4n'}-1)(2^{8n'}-1)(2^{12n'}-1) \mid 2^{16n}(2^{16n}-1)(2^{8n}+1)$, which is a contradiction.

(3). Let $K/H \cong {}^2D_8(2^{2n'})$. So, we consider $p = 2^{8n}+1 = 2^{16n'}+1$.It follows that $n = 2n'$. On the other hand, $|{}^2D_8(2^{2n'})| \mid |G|$, so $2^{112n'}(2^{16n'}-1)(2^{4n'}-1)(2^{8n'}-1)(2^{12n'}-1)(2^{16n'}-1)(2^{20n'}-1)(2^{24n'}-1)(2^{28n'}-1)(2^{32n'}-1) \mid 2^{16n}(2^{16n}-1)(2^{8n}+1)$, which is a contradiction. □

Lemma 3.4. *$K/H \cong B_a(2^b)$, $ab = 8n$; $a \geqslant 2$.*

Proof. We suppose $a = 2, 4, 8$. Let $a = 4$, so $K/H \cong B_4(2^{2n'})$. Now, we consider $p = 2^{8n'}+1$. It follows that $2^{8n}+1 = 2^{8n'}+1$, $n = n'$. On the other hand, $|B_4(2^{2n'})| \mid |G|$, but $|B_4(2^{2n'})| \nmid |B_2(2^{4n})|$, so we have a contradiction. Now, we assume $a = 8$, thus we consider $K/H \cong B_8(2^{n'})$. Now, we have $p = 2^{8n'}+1$. It follows that $2^{8n}+1 = 2^{8n'}+1$, so $n = n'$. On the other hand, $|B_8(2^n)| \nmid |B_2(2^{4n})|$, which is a contradiction. Hence, $a = 2$ and $K/H \cong B_2(2^{4n})$. Thus,$|K/H| = |B_2(2^{4n})|$. On the other hand, we know that $p \in \pi(K/H)$, so $2^{8n} + 1 = 2^{8n'} + 1$. It follows that $n = n'$. Since G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ that $H = 1$ and $G = K$. Thus, $G \cong B$, as required. $\square$

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